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Discrete-time Nonlinear Recurrent High Order Neural Sliding Mode Observer

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Abstract

This paper presents the design of an adaptive recurrent neural observer for nonlinear systems, whose mathematical model is assumed to be unknown. The observer is based on a recurrent high order neural network (RHONN), which estimates the state vector of the unknown plant dynamics. The observer presented in this paper has a sliding mode observer structure. The learning algorithm for the RHONN is based on an Extended Kalman Filter (EKF). This paper also includes the respective stability analysis, on the basis of the Lyapunov approach, for the neural observer trained with the extended Kalman filter. Simulation results are included to illustrate the applicability of the proposed scheme.

Keywords

Sliding mode, Discrete-time systems, Recurrent high order neural network, Extended Kalman filtering, Nonlinear observer.

I. Introduction

Most of the publications on nonlinear control use the assumption that all the system states are measurable. In practice, however, only some of these states are measured directly. For this reason, nonlinear state estimation is a very important topic on nonlinear system theory [10]. A nonlinear observer is a dynamic system with the same inputs and measurements as the original one; it provides an estimation of the nonlinear system states.

Numerous approaches have been proposed for the design of nonlinear observers yielding many interesting results in different directions ([2], [4], [7], [8], [10] and references therein). The Lie algebras are used in [6] to construct a nonlinear observer based on an error linearization technique. The transformation of a nonlinear system into the so-called canonical form is used in [9]; such approaches can be considered as relatively simple methods. These approaches require that the system does not have uncertainties; in practice, there are external and internal uncertainties. Observers which work in presence of uncertainties have received less attention; they are called robust. There are some results using the variable structure (VS) approach to develop robust nonlinear observers [16]; an-other approach is to use the H^∞ methodology. For both of those last ones, the design process is too complex. Sliding mode observers are widely used due to their finite-time convergence, robustness with respect to uncertainties and uncertainty estimation [2].

Neural networks (NN) have grown to be a well-established methodology, which allows for solving some very difficult problems in engineering, as exemplified by their applications in control nonlinear and complex systems [10]. This applicability is based on their theoretical capability to approximate arbitrary well continuous nonlinear mappings. The most used NN structures are: *Feedforward* networks and *Recurrent* ones [12]. The last type offers a better suited tool to model and control nonlinear systems [10].

The best well-known training approach for recurrent neural networks (RNN) is the back propagation through time learning [17]. However, it is a first order gradient descent method and hence its learning speed could be very slow [17]. Recently the Extended Kalman Filter (EKF) based algorithms has been introduced to train neural networks [17]; with the EKF based algorithm, the learning convergence is improved [17]. The EKF training of neural networks, both feedforward and recurrent ones, has proven to be reliable and practical for many applications over the past ten years [17].

All the approaches to develop nonlinear observers above mentioned need a nominal mathematical model for the plant dynamics to be known, at least partially [10]. Recurrent neural network observers have been proposed; they do not require any plant model; this technique has been successfully applied to provide a good enough state estimation [8], [10], [12]; these works were developed for continuous-time systems. Nonlinear discrete-time neural observers have been seldom discussed [7].

In this paper, we propose a sliding mode neural observer (SMNO) for a class of MIMO discrete-time nonlinear system; this observer is based on a discrete-time recurrent high order neural network (RHONN), which estimates the state vector of the unknown plant. It deals with the so-called mixed uncertainties (the presence of simultaneous external and internal uncertainties) [10]. The learning algorithm for the RHONN is based on an EKF. This paper also includes the respective stability analysis, on the basis of the Lyapunov approach, for the SMNO trained with the EKF, as well as a meaningful illustrative example.

The paper is organized as follows: first some mathematical preliminaries and the RHONN model are introduced. After that the EKF-based training algorithm is analyzed; next the observer design is described, including the stability analysis of the observer scheme; then the effectiveness of the proposed observer is illustrated via simulations.

Finally, some relevant conclusions are established.

II. Mathematical preliminaries

A. Stability Definitions

This section close follows [3]. Through this paper, we use k as the step sampling, $k \in 0 \cup \mathbb{Z}^+$, $|\bullet|$ as the absolute value and, $\|\bullet\|$ as the Euclidian norm for vectors and as any adequate norm for matrices. Consider a MIMO nonlinear system:

$$x(k+1) = F(x(k), u(k)) \tag{1}$$

where $x \in \Re^n$,$u \in \Re^m$, and $F \in \Re^n \times \Re^m \to \Re^n$ is nonlinear function.

Definition 1: The system (1) is said to be forced, or to have input. In contrast the system described by an equation without explicit presence of an input u, that is

$$x(k+1) = F(x(k))$$

is said to be unforced. It can be obtained after selecting the input u as a feedback function of the state

$$u(k) = \xi(x(k)) \tag{2}$$

Such substitution eliminates u:

$$x(k+1) = F(x(k), \xi(x(k))) \tag{3}$$

and yields an unforced system (3).

Definition 2: The solution of (1) $-$ (2) is semiglobally uniformly ultimately bounded (SGUUB), if for any Ω, a compact subset of $\Re^n$ and all $x(k_0) \in \Omega$, there exists an $\varepsilon > 0$ and a number $N(\varepsilon, x(k_0))$ such that $\|x(k)\| < \varepsilon$ for all $k \geq k_0 + N$.

In other words, the solution of (1) is said to be SGUUB if, for any a priory given (arbitrarily large) bounded set Ω and any a priory given (arbitrarily small) set Ω_0, which contains $(0,0)$ as an interior point, there exists a control (2), such that every trajectory of the closed loop system starting from Ω enters the set $\Omega_0 = \{x(k) \mid \|x(k)\| < \varepsilon\}$, in a finite time and remains in it thereafter.

Theorem 1 Let $V(x(k))$ be a Lyapunov function for a discrete-time system (1), which satisfies the following properties:

$$\begin{aligned} \gamma_1(\|x(k)\|) &\leq V(x(k)) \leq \gamma_2(\|x(k)\|) \\ V(x(k+1)) - V(x(k)) &= \Delta V(x(k)) \\ &\leq -\gamma_3(\|x(k)\|) + \gamma_3(\zeta) \end{aligned}$$

where ζ is a positive constant, $\gamma_1(\bullet)$ and $\gamma_2(\bullet)$ are strictly increasing functions, and $\gamma_3(\bullet)$ is a continuous, nondecreasing function. Thus if

$$\Delta V(x) < 0 \quad \text{for} \quad \|x(k)\| > \zeta$$

then $x(k)$ is uniformly ultimately bounded, i.e. there is a time instant k_T, such that $\|x(k)\| < \zeta, \forall\ k < k_T$.

Definition 3: [5] A subset $S \in \Re^n$ is bounded if there is $r > 0$ such that $\|x\| \leq r$ for all $x \in S$.

Definition 4: [5] A signum nonlinearity $sign(x)$ is described by

$$sign(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

Definition 5: [1] In general, a non-square matrix $C \in \Re^{p\times n}$ does not have a true inverse. However, it is possible to define the pseudoinverse $C^+ \in \Re^{n\times p}$ in the Moore-Penrose sense as

$$C^+ \triangleq \left(C^\top C\right)^{-1} C^T$$

B. *Discrete-time Recurrent High Order Neural Networks*

Consider the following discrete-time recurrent high order neural network (RHONN):

$$\widehat{x}_i(k+1) = w_i^\top z_i(\widehat{x}(k), u(k)), \quad i = 1, \cdots, n \tag{4}$$

where $\widehat{x}_i$ $(i = 1, 2, \cdots, n)$ is the state of the ith neuron, L_i is the respective number of higer-order connections, $\{I_1, I_2, \cdots, I_{L_i}\}$ is a collection of non-ordered subsets of $\{1, 2, \cdots, n\}$, n is the state dimension, w_i $(i = 1, 2, \cdots, n)$

is the respective on-line adapted weight vector, and $z_i(\widehat{x}(k), u(k))$ is given by

$$z_i(x(k), u(k)) = \begin{bmatrix} z_{i_1} \\ z_{i_2} \\ \vdots \\ z_{i_{L_i}} \end{bmatrix} = \begin{bmatrix} \Pi_{j\in I_1} y_{i_j}^{d_{ij}(1)} \\ \Pi_{j\in I_2} y_{i_j}^{d_{ij}(2)} \\ \vdots \\ \Pi_{j\in I_{L_i}} y_{i_j}^{d_{ij}(L_i)} \end{bmatrix} \tag{5}$$

with $d_{j_i}(k)$ being a nonnegative integers, and y_i is defined as follows:

$$y_i = \begin{bmatrix} y_{i_1} \\ \vdots \\ y_{i_1} \\ y_{i_{n+1}} \\ \vdots \\ y_{i_{n+m}} \end{bmatrix} = \begin{bmatrix} S(\widehat{x}_1) \\ \vdots \\ S(\widehat{x}_n) \\ S(u_1) \\ \vdots \\ S(u_m) \end{bmatrix} \tag{6}$$

In (6), $u = [u_1, u_2, \ldots, u_m]^\top$ is the input vector to the neural network, and $S(\bullet)$ is defined by

$$S(x) = \frac{1}{1 + \exp(-\beta\widehat{x})} \tag{7}$$

Consider the problem to approximate the general discrete-time nonlinear system (1), by the following discrete-time RHONN parallel representation, which is supposed observable [13]:

$$x_i\,(k+1) = w_i^{*\top} z_i\,(x(k), u(k)) + \epsilon_{z_i} \tag{8}$$

where x_i is the ith plant state, ϵ_{z_i} is a bounded approximation error, which can be reduced by increasing the number of the adjustable weights [13]. Assume that there exists ideal weights vector w_i^* such that $\|\epsilon_{z_i}\|$ can be minimized on a compact set $\Omega_{z_i} \subset \Re^{L_i}$. The ideal weight vector w_i^* is an artificial quantity required for analytical purpose [13]. In general it is assumed that this vector exists and is constant but unknown. Let us define its estimate as w_i and the estimation error as

$$\widetilde{w}_i\,(k) = w_i\,(k) - w_i^* \tag{9}$$

The estimate w_i is used for stability analysis, which will be discussed later. Since w_i^* is constant, then $\widetilde{w}_i\,(k+1) - \widetilde{w}_i\,(k) = w_i\,(k+1) - w_i\,(k)$, $\forall k \in 0 \cup \mathbb{Z}^+$.

III. The EKF training algorithm

It is known that Kalman filtering (KF) estimates the state of a linear system with additive state and output white noises [4], [14]. For KF-based neural network training, the network weights become the states to be estimated. In this case the error between the neural network output and the measured plant output can be considered as additive white noise. Due to the fact that the neural network mapping is nonlinear, an EKF-type is required (see [11] and references therein). The training goal is to find the optimal weight values which minimize the prediction error. In this paper we use a decupled EKF-based training algorithm is described by:

$$\begin{aligned} w_i(k+1) &= w_i(k) + \eta_i K_i(k) e(k) \\ K_i(k) &= P_i(k) H_i(k) M_i(k) \\ P_i(k+1) &= P_i(k) - K_i(k) H_i^\top(k) P_i(k) + Q_i(k) \\ i &= 1, \cdots, n \end{aligned} \tag{10}$$

with

$$M_i(k) = \left[R_i(k) + H_i^\top(k) P_i(k) H_i(k)\right]^{-1} \tag{11}$$

$$e(k) = y(k) - \widehat{y}(k) \tag{12}$$

where $e(k) \in \Re^p$ is the observation error, $P_i(k) \in \Re^{L_i \times L_i}$ is the prediction error covariance matrix at step k, $w_i \in \Re^{L_i}$ is the weight (state) vector, L_i is the respective number of neural network weights, $y \in \Re^p$ is the plant output, $\widehat{y} \in \Re^p$ is the observer output, n is the number of states, $K_i \in \Re^{L_i \times p}$ is the Kalman gain matrix, $Q_i \in \Re^{L_i \times L_i}$ is the NN weight estimation noise covariance matrix, $R_i \in \Re^{p \times p}$ is the error noise covariance; $H_i \in \Re^{L_i \times p}$ is a matrix, in which each entry (H_{ij}) is the derivative of the neural output, with respect to one neural network weight, (w_{ij}), as follows

$$H_{ij}(k) = \left[\frac{\partial \widehat{y}(k)}{\partial w_{i_j}(k)}\right]^\top_{w_i(k)=w_i(k+1)} \tag{13}$$

where $i = 1, ..., n$ and $j = 1, ..., L_i$. Usually $P_i$and Q_i are initialized as diagonal matrices, with entries $P_i(0)$ and $Q_i(0)$, respectively. It is important to remark that $H_i(k)$, $K_i(k)$ and $P_i(k)$ for the EKF are bounded; for a detailed explanation of this fact see [14].

IV. Neural observer design

To this end, we design the sliding mode neural network observer. In [15], a sliding mode observer is proposed for linear time-invariant systems; it has a term to estimate the state vector and other one selected as a sliding mode in order to enforce $\widetilde{y} = y - \widehat{y} = 0$; the mismatch between the output vector y and its estimate $\widehat{y}$ is reduced to zero [15]. In this, paper we use this structure to construct a sliding mode observer for nonlinear time-variant systems on the basis of a RHONN.

In this section, we consider to estimate the state of a discrete-time nonlinear system, which is assumed to be observable, given by

$$\begin{aligned} x(k+1) &= F(x(k), u(k)) + d(k) \\ y(k) &= Cx(k) \end{aligned} \tag{14}$$

where $x \in \Re^n$ is the state vector of the system, $u(k) \in \Re^m$ is the input vector, $y(k) \in \Re^p$ is the output vector, $C \in \Re^{p \times n}$ is a known output matrix, $d(k) \in \Re^n$ is a disturbance vector and $F(\bullet)$ is a smooth vector field and $F_i(\bullet)$ its entries; hence (14) can be rewritten as:

$$\begin{aligned} x(k) &= \begin{bmatrix} x_1(k) & \ldots & x_i(k) & \ldots & x_n(k) \end{bmatrix}^\top \\ d(k) &= \begin{bmatrix} d_1(k) & \ldots & d_i(k) & \ldots & d_n(k) \end{bmatrix}^\top \\ x_i(k+1) &= F_i(x(k), u(k)) + d_i(k) \quad , \quad i = 1, \cdots, n \\ y(k) &= Cx(k) \end{aligned} \tag{15}$$

For system (15), we propose a sliding mode neural observer (SMNO) with the following structure:

$$\begin{aligned} \widehat{x}(k) &= \begin{bmatrix} \widehat{x}_1(k) & \ldots & \widehat{x}_i(k) & \ldots & \widehat{x}_n(k) \end{bmatrix}^\top \\ \widehat{x}_i(k+1) &= w_i^\top z_i(\widehat{x}(k), u(k)) + L_i sign(e(k)) \\ \widehat{y}(k) &= C\widehat{x}(k) \\ i &= 1, \cdots, n \end{aligned} \tag{16}$$

with $L_i \in \Re^p$, w_i and z_i as in (4); the weight vectors are updated on-line with a decupled EKF $(10) - (13)$ and the output error is defined by

$$e(k) = y(k) - \widehat{y}(k) \tag{17}$$

and the state estimation error as:

$$\widetilde{x}(k) = x(k) - \widehat{x}(k) \tag{18}$$

Before proceeding to demonstrate the main result of this paper, we need to establish the following two lemmas.

Lemma 2. (17) can be formulated as

$$e(k+1) = e(k) + \Delta e(k) \tag{19}$$

with $\Delta e(k) \leq -\gamma_i e(k)$ and $\gamma_i = \max \left\| \eta_i H_i^\top(k) K_i(k) \right\|$.

Proof: From (17), we obtain

$$\frac{\partial e(k)}{\partial w_i(k)} = -\frac{\partial \widehat{y}(k)}{\partial w_i(k)} \tag{20}$$

Let us approximate (20) by

$$\Delta e(k) = \left[\frac{\partial e(k)}{\partial w_i(k)} \right]^\top \Delta w_i(k) \tag{21}$$

Substituting (13), (17) and (20) in (21), yields

$$\Delta e(k) = -\eta_i H_i^\top(k) K_i(k) e(k) \tag{22}$$

Defining

$$\gamma_i = \max \left\| \eta_i H_i^\top(k) K_i(k) \right\|$$

with $M_i(k)$ as in (11), (22)can be rewritten as

$$\Delta e(k) \leq -\gamma_i e(k) \tag{23}$$

■

Lemma 3. The estimation weight error (9) can be written as:

$$\widetilde{w}_i^\top(k) = \left[\epsilon_i^{'}(k) - L_i sign(e(k)) - C_i^{+} e(k) - C_i^{+} \Delta e(k) \right] z_i^{+}(x(k), u(k))$$

with C_i^{+} the ith row of C^{+} and $\epsilon_i^{'}(k) = \epsilon_i + d_i(k)$.

Proof: From $(15)-(17)$, we have

$$\begin{aligned} e(k) &= y(k) - \widehat{y}(k) = C(x(k) - \widehat{x}(k)) \qquad (24) \\ e(k+1) &= C(x(k+1) - \widehat{x}(k+1)) \\ &= C\left(w^{*\top} z(x(k), u(k)) + \epsilon + d(k) - w^\top(k) z(\widehat{x}(k), u(k)) - L sign(e(k))\right) \\ &= e(k) + \Delta e(k) \end{aligned}$$

then

$$\widetilde{w}^{\top}(k)\, z\,(x(k), u(k)) = \epsilon + d\,(k) - Lsign\,(e\,(k)) - C^{+}\,(e\,(k) + \Delta e\,(k))$$

and

$$\widetilde{w}^{\top}(k) = \left[\epsilon + d\,(k) - Lsign\,(e\,(k)) - C^{+}e\,(k) - C^{+}\Delta e\,(k)\right] z^{+}\,(x(k), u(k))$$

Then, for the ith element, we have

$$\widetilde{w}_i^{\top}(k) = \left[\epsilon_i^{'}(k) - L_i sign\,(e\,(k)) - C_i^{+}e\,(k) - C_i^{+}\Delta e\,(k)\right] z_i^{+}\,(x(k), u(k)) \tag{25}$$

■

Considering $(10) - (17)$, we establish the main result in the following theorem.

Theorem 2: For the system (15), the SMNO (16) trained with the EKF-based algorithm (10), ensures that the output error (17) and the estimation error (18) are semiglobally uniformly ultimately bounded (SGUUB); moreover, the SMNO weights remain bounded.

Proof: Consider the Lyapunov function candidate, for $e\,(k)$, $w\,(k)$, defined as

$$V_i\,(k) = e^{\top}(k)\, e\,(k) + \widetilde{w}_i^{\top}(k)\, \widetilde{w}_i\,(k) \tag{26}$$

whose first difference is:

$$\begin{aligned} \Delta V_i\,(k) &= V_i\,(k+1) - V_i\,(k) \\ &= e^{\top}(k+1)\, e\,(k+1) + \widetilde{w}_i^{\top}(k+1)\, \widetilde{w}_i\,(k+1) - e^{\top}(k)\, e\,(k) - \widetilde{w}_i^{\top}(k)\, \widetilde{w}_i\,(k) \end{aligned} \tag{27}$$

From (9)and (10), then

$$\widetilde{w}_i\,(k+1) = \widetilde{w}_i\,(k) + \eta_i K_i\,(k)\, e\,(k) \tag{28}$$

Let us define

$$\begin{aligned} &\left[\widetilde{w}_i\,(k) + \eta_i K_i\,(k)\, e\,(k)\right]^{\top} \left[\widetilde{w}_i\,(k) + \eta_i K_i\,(k)\, e\,(k)\right] \\ =\; &\widetilde{w}_i^{\top}(k)\, \widetilde{w}_i\,(k) + 2\eta_i \widetilde{w}_i^{\top}(k)\, K_i\,(k)\, e\,(k) + (\eta_i K_i\,(k)\, e\,(k))^{\top} (\eta_i K_i\,(k)\, e\,(k)) \end{aligned} \tag{29}$$

From (17) ,then

$$e\,(k+1) = e\,(k) + \Delta e\,(k)$$

$$e^{\top}(k+1)\, e\,(k+1) = e^{\top}(k)\, e\,(k) + e^{\top}(k)\, \Delta e\,(k) + \Delta e^{\top}(k)\, e\,(k) + \Delta e^{\top}(k)\, \Delta e\,(k)$$

$$e^{\top}(k+1)\, e\,(k+1) - e^{\top}(k)\, e\,(k) = e^{\top}(k)\, \Delta e\,(k) + \Delta e^{\top}(k)\, e\,(k) + \Delta e^{\top}(k)\, \Delta e\,(k) \tag{30}$$

Using (29), (30) and Lemma 3 in (27):

$$\begin{aligned}\Delta V_i(k) &= e^\top(k)\Delta e(k) + \Delta e^\top(k) e(k) + \Delta e^\top(k)\Delta e(k)\\ &\quad +2\eta_i\left[\epsilon_i'(k) - L_i sign(e(k)) - C_i^+ e(k) - C_i^+\Delta e(k)\right] z_i^+(k)) K_i(k) e(k)\\ &\quad +(\eta_i K_i(k) e(k))^\top \eta_i K_i(k) e(k)\end{aligned} \tag{31}$$

From Lemma 2, substituting (23) , in (31), then

$$\begin{aligned}\Delta V_i(k) &\leq -2\gamma_i e^\top(k) e(k) + \gamma_i^2 e^\top(k) e(k) \\ &\quad +2\eta_i\left[\epsilon_i'(k) - L_i sign(e(k)) - C_i^+ e(k) + \eta_i\gamma_i C_i^+ e(k)\right] z_i^+(k)) K_i(k) e(k)\\ &\quad +(\eta_i K_i(k) e(k))^\top \eta_i K_i(k) e(k)\end{aligned} \tag{32}$$

Defining $\delta_i = \eta_i\gamma_i C_i^+ - C_i^+$, (32) can be written as

$$\begin{aligned}\Delta V_i(k) &\leq -2\gamma_i e^\top(k) e(k) + \gamma_i^2 e^\top(k) e(k) + 2\eta_i\left[\epsilon_i'(k) + \delta_i e(k) - L_i sign(e(k))\right] z_i^+(k)) K_i(k) e(k)\\ &\quad +(\eta_i K_i(k) e(k))^\top \eta_i K_i(k) e(k)\\ &\leq -2\gamma_i e^\top(k) e(k) + \gamma_i^2 e^\top(k) e(k) + 2\eta_i\epsilon_i'(k) z_i^+(k)) K_i(k) e(k)\\ &\quad +2\eta_i\delta_i e(k) z_i^+(k)) K_i(k) e(k) - 2\eta_i L_i sign(e(k)) z_i^+(k)) K_i(k) e(k) + (\eta_i K_i(k) e(k))^\top \eta_i K_i(k) e(k)\end{aligned}$$

and with $\nu_i(k) = \left\|\epsilon_i'(k)\right\| - \|L_i\|$ we have

$$\begin{aligned}\Delta V_i(k) &\leq -2\gamma_i\|e(k)\|^2 + \gamma_i^2\|e(k)\|^2 + 2\eta_i|\nu_i(k)|\left\|z_i^+(k)) K_i(k)\right\|\|e(k)\|\\ &\quad +2\eta_i\|\delta_i\|\left\|z_i^+(k)) K_i(k)\right\|\|e(k)\|^2 + \eta_i\|K_i(k)\|^2\|e(k)\|^2\end{aligned}$$

then, there exits $\eta_i > 0$ and L_i such that

$$\Delta V_i(k) < 0, \quad \text{for} \quad \|e(k)\| > \kappa_i \tag{33}$$

with

$$\kappa_i = \frac{2\eta_i \upsilon_i\left\|z_i^+ K_i\right\|}{2\gamma_i - \gamma_i^2 - 2\eta_i\|\delta_i\|\left\|z_i^+ K_i\right\| - \eta_i^2\|K_i\|^2}$$

From (33) it follows, the boundness of $V_i(k)$ for a $k \geq k_T$, which leads the SGUUB of $e(k)$ and $\widetilde{w}_i(k)$. Considering (18) and (24) , it is easy too see that the estimation error has an algebraic relation with $e(k)$. For that reason if $e(k)$ is bounded $\widetilde{x}(k)$ is bounded too.

$$\begin{aligned}\widetilde{x}(k) &= C^+ e(k)\\ \|\widetilde{x}(k)\| &\leq \left\|C^+\right\|\|e(k)\|\end{aligned}$$

Using (25) and (33), it is easy too see that the estimation weight error (9) has an algebraic relation with $e(k)$ and $z_i^+(x(k), u(k))$. For that reason if $e(k)$ is bounded and given that $z_i^+(x(k), u(k))$, as defined in (5), is bounded, then $\widetilde{w}(k)$ is bounded too.

$$
\begin{aligned}
\widetilde{w}_i^\top(k) &= \left[\epsilon_i^{'}(k) - L_i sign(e(k)) - C_i^+ e(k) - C_i^+ \Delta e(k)\right] z_i^+(x(k), u(k)) \\
\widetilde{w}_i^\top(k) &\leq \left|\epsilon_i^{'}(k) - L_i sign(e(k)) - C_i^+ e(k) - C_i^+ \Delta e(k)\right| \left\|z_i^+(x(k), u(k))\right\|
\end{aligned}
$$

■

V. Simulation results

In this section, the neural observer is applied to a modified Van der Pol oscillator, whose nonlinear dynamics is represented by the following equation [18]:

$$
\begin{aligned}
x_1(k+1) &= x_1(k) + T x_2(k) + d_1(k) \\
x_2(k+1) &= x_2(k) + T\left(-\xi\left(x_1^2(k) - 1\right) x_2(k)\right) + T\left(-x_1(k) + u(k)\right) + d_2(k) \\
y(k) &= x_1(k) \\
d_1(k) &= 0.1\sin(k) \\
d_2(k) &= 0.1\cos(k)
\end{aligned} \tag{34}
$$

where variables $x \in \Re^2$, $u \in \Re$, and $y \in \Re$ are the state, input, and output of the system, respectively; $d_1(k)$ and $d_2(k)$ are bounded external disturbances; T is the sampling period, which is fixed at $0.1s$ and ξ is a parameter which nominal value is equal to 2.

To estimate the state x_2 we use the SMNO (16) with $n = 2$ trained with the EKF (10).

$$
\begin{aligned}
\widehat{x}_1(k+1) &= w_{11}(k) S^2(\widehat{x}_1(k)) + w_{12}(k) S(\widehat{x}_1(k)) S(\widehat{x}_2(k)) + w_{13}(k) S^2(\widehat{x}_2(k)) \\
\widehat{x}_2(k+1) &= w_{21}(k) S^2(\widehat{x}_1(k)) + w_{22}(k) S(\widehat{x}_1(k)) S(\widehat{x}_2(k)) + w_{23}(k) S^2(\widehat{x}_2(k)) + w_{24}(k) S^3(u(k)) \\
\widehat{y}(k) &= \widehat{x}_1(k) \\
u(k) &= \cos\left(\frac{2\pi k}{25}\right)
\end{aligned} \tag{35}
$$

The training is performed on-line, using a parallel configuration as displayed in Fig. 1. All the NN states are initialized in a random way. The covariances matrices are initialized as diagonals, with nonzero elements: $P_1(0) = P_2(0) = 10000$; $Q_1(0) = Q_2(0) = 500$ and $R_1(0) = R_2(0) = 10000$, respectively. The simulation results are

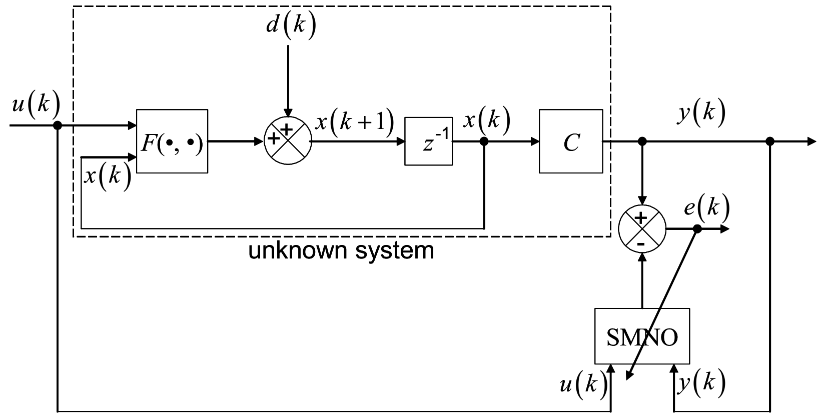

Fig. 1. Observation scheme

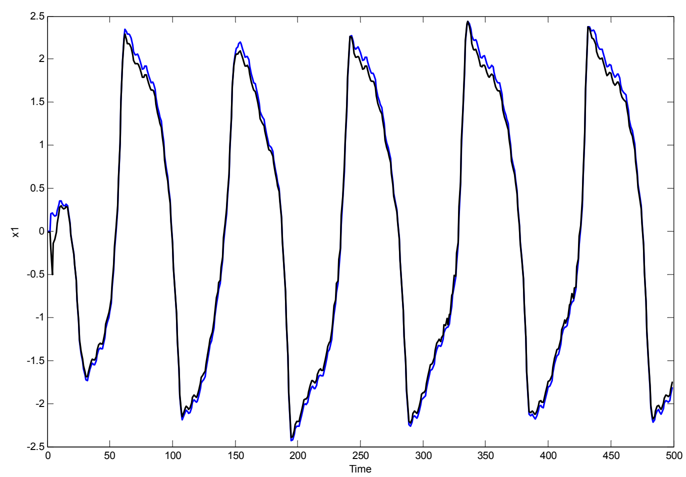

Fig. 2. Time evolution of the state $x_1(k)$ (solid line) and its estimated $\widehat{x}_1(k)$ (dashed line)

presented in Fig. 2, and Fig. 3. They display the time evolution of the estimated states $x_1(k)$ and $x_2(k)$, respectively. Fig. 4. shows the estimation errors. Fig. 5 displays the parametric variation for ξ increment, and Fig. 6 shows the bounded external disturbances.

VI. Conclusions

A RHONN is used to design a SMNO for a class of MIMO discrete-time nonlinear system in presence of both external and internal uncertainties. The SMNO proposed is trained using an EKF-based algorithm. The training of the SMNO is performed on-line for a parallel configuration. The boundness of the output and estimation errors is established on the basis of the Lyapunov approach. Simulation results shows the effectiveness of the proposed SMNO. The importance of the results presented in this paper are based on the necessity of observers for unknown

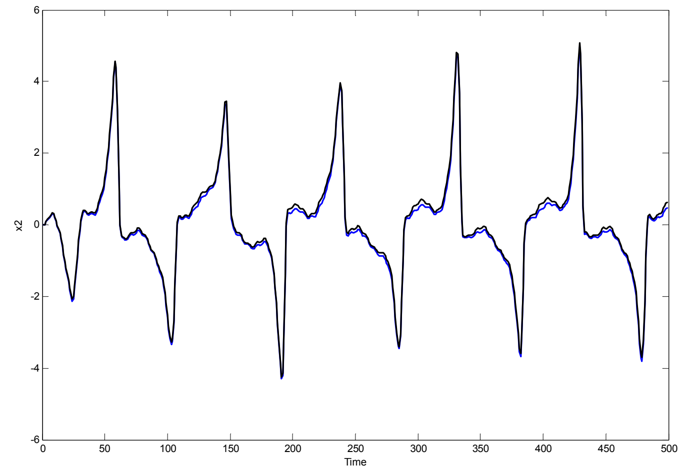

Fig. 3. Time evolution of the state $x_2(k)$ (solid line) and its estimated $\widehat{x}_2(k)$ (dashed line)

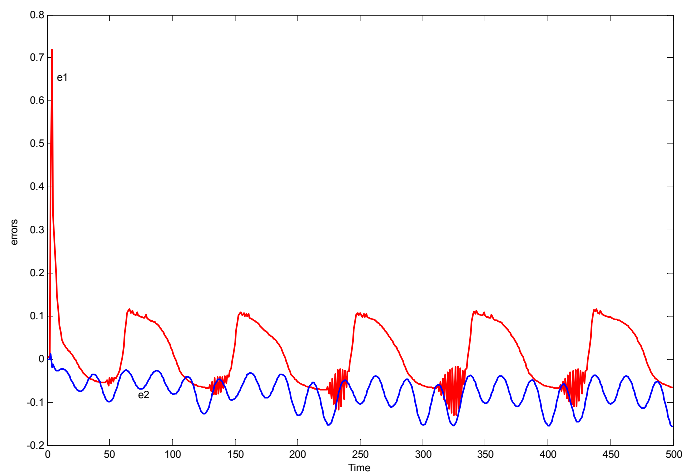

Fig. 4. Etimation errors $\widetilde{x}_1(k)$ (dashed line) and $\widetilde{x}_2(k)$ (solid line).

or partially unknown discrete-time nonlinear systems.

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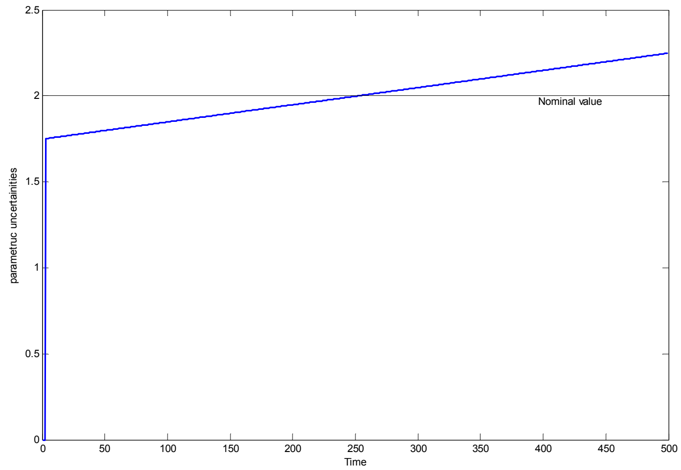

Fig. 5. Uncertainities in parameter ξ

Fig. 6. Disturbances $d_1(k)$ (solid line) and $d_2(k)$ (dashed line)

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