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Discrete-time Nonlinear Recurrent High Order Neural Observer

Alma Y. Alanis, Edgar N. Sanchez and Alexander G. Loukianov
CINVESTAV, Unidad Guadalajara, Apartado Postal 31-438, Plaza La Luna, Guadalajara,
Jalisco, C.P. 45091, Mexico, e-mail:sanchez@gdl.cinvestav.mx

Abstract— **This paper presents the design of an adaptive recurrent neural observer for nonlinear systems, whose mathematical model is assumed to be unknown. The observer is based on a recurrent high order neural network (RHONN), which estimates the state vector of the unknown plant dynamics. The learning algorithm for the RHONN is based on an extended Kalman filter. This paper also includes the respective stability analysis, on the basis of the Lyapunov approach, for the neural observer trained with the extended Kalman filter. Some simulation results are included to illustrate the applicability of the proposed scheme.**

Keywords— **Discrete-time systems, Recurrent high order neural observer, Exended Kalman filtering, Nonlinear observer.**

I. Introduction

Most of the nonlinear control publications assume the complete accessibility of system states; this is not always possible. For that reason the solution of the nonlinear state estimation problem is a very important topic for nonlinear control [9].

The state estimation problem has received much attention by many authors, who have obtained many interesting results in different directions. Many of those results need the use of a special nonlinear transformation [8] or a linearization technique [3], [5]; such approaches can be considered as a relatively simple method to construct nonlinear observers. However, they do not consider uncertainties. In practice, we work in presence of external and internal uncertainties.

Other kind of observers which have a good performance even in presence of model and disturbance uncertainties, are called robust; their design process is too complex [14]. All the approaches mentioned above need the previous knowledge of the plant dynamics. Recently, other kind of observer has emerged: neural observers [9], [11], [7], [6], for unknown plant dynamics. For linear unknown systems, observer design has been widely investigated [11]. For nonlinear systems there are few results; besides, most of those works were developed for continuous-time systems; the nonlinear discrete-time case have not been discussed to the same degree.

The best well-known training approach for recurrent neural networks (RNN) is the back propagation through time learning [15]. However, it is a first order gradient descent method and hence its learning speed could be very slow [15]. Recently Extended Kalman Filter (EKF) based algorithms has been introduced to train neural networks to improve the learning convergence [15]. The EKF training of neural networks, both feedforward and recurrent ones, has proven to be reliable and practical for many applications over the past ten years [15].

In this paper, we consider a class of MIMO discrete-time nonlinear system, for which we develop a Luenberger-like observer [9]. This observer is based on a recurrent high order neural network (RHONN), which estimates the state vector of the unknown plant dynamics. It deals with the so-called mixed uncertainties (the presence of simultaneous external and internal uncertainties) [9]. The learning algorithm for the RHONN is based on an extended Kalman filter. This paper also includes the respective stability analysis, on the basis of the Lyapunov approach, for the neural observer trained with the extended Kalman filter.

The paper is organized as follows: first some mathematical preliminaries and the RHONN model are introduced. After that the EKF-based training algorithm is analyzed; next the observer design is treated including the stability analysis; then the effectiveness of the proposed observer is illustrated via simulations. Finally, some relevant conclusions are established.

II. Mathematical preliminaries

A. Stability Definitions

This section close follows [2]. Through this paper, we use k as the step sampling, $k \in 0 \cup \mathbb{Z}^+$, $|\bullet|$ as the absolute value and, $\|\bullet\|$ as the Euclidian norm for vectors and as any adequate norm for matrices. Consider a MIMO nonlinear system:

$$\chi(k+1) = F(\chi(k), u(k)) \tag{1}$$

where $\chi \in \Re^n, u \in \Re^m$, and $F \in \Re^n \times \Re^m \rightarrow \Re^n$ is nonlinear function.

Definition 1: The system (1) is said to be forced, or to have input. In contrast the system described by an equation without explicit presence of an input u, that is

$$\chi(k+1) = F(\chi(k))$$

is said to be unforced. It can be obtained after selecting the input u as a feedback function of the state

$$u(k) = \xi(\chi(k)) \tag{2}$$

Such substitution eliminates u:

$$\chi(k+1) = F(\chi(k), \xi(\chi(k))) \tag{3}$$

and yields an unforced system (3).

Definition 2: The solution of (1) – (2) is semiglobally uniformly ultimately bounded (SGUUB), if for any Ω, a compact subset of $\Re^n$ and all $\chi(k_0) \in \Omega$, there exists an $\varepsilon > 0$ and a number $N(\varepsilon, \chi(k_0))$ such that $\|\chi(k)\| < \varepsilon$ for all $k \geq k_0 + N$.

In other words, the solution of (1) is said to be SGUUB if, for any a priory given (arbitrarily large) bounded set Ω and any a priory given (arbitrarily small) set Ω_0, which contains $(0,0)$ as an interior point, there exists a control (2), such that every trayectory of the closed loop system starting from Ω enters the set $\Omega_0 = \{\chi(k) \mid \|\chi(k)\| < \varepsilon\}$, in a finite time and remains in it thereafter.

Theorem 1 Let $V(\chi(k))$ be a Lyapunov function for a discrete-time system (1), which satisfies the following properties:

$$\begin{aligned}
\gamma_1(\|\chi(k)\|) &\leq V(\chi(k)) \leq \gamma_2(\|\chi(k)\|) \\
V(\chi(k+1)) - V(\chi(k)) &= \Delta V(\chi(k)) \\
&\leq -\gamma_3(\|\chi(k)\|) + \gamma_3(\zeta)
\end{aligned}$$

where ζ is a positive constant, $\gamma_1(\bullet)$ and $\gamma_2(\bullet)$ are strictly increasing functions, and $\gamma_3(\bullet)$ is a continuous, nondecreasing function. Thus if

$$\Delta V(\chi) < 0 \quad \text{for} \quad \|\chi(k)\| > \zeta$$

then $\chi(k)$ is uniformly ultimately bounded, i.e. there is a time instant k_T, such that $\|\chi(k)\| < \zeta, \forall\ k < k_T$.

Definition 3: A subset $S \in \Re^n$ is bounded if there is $r > 0$ such that $\|\chi\| \leq r$ for all $\chi \in S$.

Definition 4: [1] In general, a non-square matrix $C \in \Re^{p \times n}$ does not have a true inverse. However, it is possible to define the pseudoinverse $C^+ \in \Re^{n \times p}$ in the Moore-Penrose sense as

$$C^+ \triangleq \left(C^\top C\right)^{-1} C^T$$

B. Discrete-time Recurrent High Order Neural Networks

Consider the following discrete-time recurrent high order neural network (RHONN):

$$\widehat{x}_i(k+1) = w_i^\top z_i(\widehat{x}(k), u(k)), \quad i = 1, \cdots, n \tag{4}$$

where $\widehat{x}_i$ $(i = 1, 2, \cdots, n)$ is the state of the ith neuron, L_i is the respective number of higer-order connections, $\{I_1, I_2, \cdots, I_{L_i}\}$ is a collection of non-ordered subsets of $\{1, 2, \cdots, n\}$, n is the state dimension, w_i $(i = 1, 2, \cdots, n)$ is the respective on-line adapted weight vector, and $z_i(\widehat{x}(k), u(k))$ is given by

$$z_i(x(k), u(k)) = \begin{bmatrix} z_{i_1} \\ z_{i_2} \\ \vdots \\ z_{i_{L_i}} \end{bmatrix} = \begin{bmatrix} \Pi_{j \in I_1} y_{i_j}^{d_{ij}(1)} \\ \Pi_{j \in I_2} y_{i_j}^{d_{ij}(2)} \\ \vdots \\ \Pi_{j \in I_{L_i}} y_{i_j}^{d_{ij}(L_i)} \end{bmatrix} \tag{5}$$

with $d_{j_i}(k)$ being a nonnegative integers, and y_i is defined as follows:

$$y_i = \begin{bmatrix} y_{i_1} \\ \vdots \\ y_{i_1} \\ y_{i_{n+1}} \\ \vdots \\ y_{i_{n+m}} \end{bmatrix} = \begin{bmatrix} S(\widehat{x}_1) \\ \vdots \\ S(\widehat{x}_n) \\ S(u_1) \\ \vdots \\ S(u_m) \end{bmatrix} \tag{6}$$

In (6), $u = [u_1, u_2, \ldots, u_m]^\top$ is the input vector to the neural network, and $S(\bullet)$ is defined by

$$S(x) = \frac{1}{1 + \exp(-\beta \widehat{x})} \tag{7}$$

Consider the problem to approximate the general discrete-time nonlinear system (1), by the following discrete-time RHONN parallel representation, which is supposed observable [12]:

$$x_i(k+1) = w_i^{*\top} z_i(x(k), u(k)) + \epsilon_{z_i} \tag{8}$$

where x_i is the ith plant state, ϵ_{z_i} is a bounded approximation error, which can be reduced by increasing the number of the adjustable weights [12]. Assume that there exists ideal weights vector w_i^* such that $\|\epsilon_{z_i}\|$ can be minimized on a compact set $\Omega_{z_i} \subset \Re^{L_i}$. The ideal weight vector w_i^* is an artificial quantity required for analytical purpose [12]. In general it is assumed that this vector exists and is constant but unknown. Let us define its estimate as w_i and the estimation error as

$$\widetilde{w}_i(k) = w_i(k) - w_i^* \tag{9}$$

The estimate w_i is used for stability analysis, which will be discussed later. Since w_i^* is constant, then $\widetilde{w}_i(k+1) - \widetilde{w}_i(k) = w_i(k+1) - w_i(k)$, $\forall k \in 0 \cup \mathbb{Z}^+$.

III. The EKF training algorithm

It is known, that Kalman filtering (KF) estimates the state of a linear system with additive state and output white noises [3], [13]. For KF-based neural network training, the network weights become the states to be estimated. In this case the error between the neural network output and the measured plant output can be considered as additive white noise. Due to the fact that the neural network mapping is nonlinear, an EKF-type is required (see [10] and references therein). The training goal is to find the optimal weight values which minimize the prediction error. In this paper we use a decupled EKF-based training algorithm is described by:

$$\begin{aligned}
w_i(k+1) &= w_i(k) + \eta_i K_i(k) e(k) \qquad (10) \\
K_i(k) &= P_i(k) H_i(k) M_i(k) \\
P_i(k+1) &= P_i(k) - K_i(k) H_i^\top(k) P_i(k) + Q_i(k) \\
i &= 1, \cdots, n
\end{aligned}$$

with

$$
\begin{aligned}
M_i(k) &= \left[R_i(k) + H_i^\top(k)\, P_i(k)\, H_i(k)\right]^{-1} && (11)\\
e(k) &= y(k) - \widehat{y}(k) && (12)
\end{aligned}
$$

where $e(k) \in \Re^p$ is the observation error, $P_i(k) \in \Re^{L_i \times L_i}$ is the prediction error covariance matrix at step k, $w_i \in \Re^{L_i}$ is the weight (state) vector, L_i is the respective number of neural network weights, $y \in \Re^p$ is the plant output, $\widehat{y} \in \Re^p$ is the observer output, n is the number of states, $K_i \in \Re^{L_i \times p}$ is the Kalman gain matrix, $Q_i \in \Re^{L_i \times L_i}$ is the NN weight estimation noise covariance matrix, $R_i \in \Re^{p \times p}$ is the error noise covariance; $H_i \in \Re^{L_i \times p}$ is a matrix, in which each entry (H_{ij}) is the derivative of the neural output, with respect to one neural network weight, (w_{ij}), as follows

$$
H_{ij}(k) = \left[\frac{\partial \widehat{y}(k)}{\partial w_{i_j}(k)}\right]^\top_{w_i(k) = w_i(k+1)} \tag{13}
$$

where $i = 1, ..., n$ and $j = 1, ..., L_i$. Usually $P_i$and Q_i are initialized as diagonal matrices, with entries $P_i(0)$ and $Q_i(0)$, respectively. It is important to remark that $H_i(k)$, $K_i(k)$ and $P_i(k)$ for the EKF are bounded; for a detailed explanation of this fact see [13].

IV. Observer design

In this section, we consider to estimate the state of a discrete-time nonlinear system, which is assumed to be observable, given by

$$
\begin{aligned}
x(k+1) &= F(x(k), u(k)) + d(k)\\
y(k) &= Cx(k) && (14)
\end{aligned}
$$

where $x \in \Re^n$ is the state vector of the system, $u(k) \in \Re^m$ is the input vector, $y(k) \in \Re^p$ is the output vector, $C \in \Re^{p \times n}$ is a known output matrix, $d(k) \in \Re^n$ is a disturbance vector and $F(\bullet)$ is a smooth vector field and $F_i(\bullet)$ its entries; hence (14) can be rewritten as:

$$
\begin{aligned}
x(k) &= \begin{bmatrix} x_1(k) & \ldots & x_i(k) & \ldots & x_n(k) \end{bmatrix}^\top\\
d(k) &= \begin{bmatrix} d_1(k) & \ldots & d_i(k) & \ldots & d_n(k) \end{bmatrix}^\top\\
x_i(k+1) &= F_i(x(k), u(k)) + d_i(k) \quad , \quad i = 1, \cdots, n\\
y(k) &= Cx(k) && (15)
\end{aligned}
$$

For system (15), we propose a Luenberger neural observer (SMNO) with the following structure:

$$
\begin{aligned}
\widehat{x}(k) &= \begin{bmatrix} \widehat{x}_1(k) & \ldots & \widehat{x}_i(k) & \ldots & \widehat{x}_n(k) \end{bmatrix}^\top\\
\widehat{x}_i(k+1) &= w_i^\top z_i(\widehat{x}(k), u(k)) + L_i e(k)\\
\widehat{y}(k) &= C\widehat{x}(k)\\
i &= 1, \cdots, n && (16)
\end{aligned}
$$

with $L_i \in \Re^p$, w_i and z_i as in (4); the weight vectors are updated on-line with a decupled EKF $(10) - (13)$ and the output error is defined by

$$
e(k) = y(k) - \widehat{y}(k) \tag{17}
$$

and the state estimation error as:

$$
\widetilde{x}(k) = x(k) - \widehat{x}(k) \tag{18}
$$

Before proceeding to demonstrate the main result of this paper, we need to establish the following two lemmas.

Lemma 2. (17) can be formulated as

$$
e(k+1) = e(k) + \Delta e(k) \tag{19}
$$

with $\Delta e(k) \leq -\gamma_i e(k)$ and $\gamma_i = \max \left\| \eta_i H_i^\top(k)\, K_i(k) \right\|$.

Proof: From (17), we obtain

$$
\frac{\partial e(k)}{\partial w_i(k)} = -\frac{\partial \widehat{y}(k)}{\partial w_i(k)} \tag{20}
$$

Let us approximate (20) by

$$
\Delta e(k) = \left[\frac{\partial e(k)}{\partial w_i(k)}\right]^\top \Delta w_i(k) \tag{21}
$$

Substituting (13), (17) and (20) in (21), yields

$$
\Delta e(k) = -\eta_i H_i^\top(k)\, K_i(k)\, e(k) \tag{22}
$$

Defining

$$
\gamma_i = \max \left\| \eta_i H_i^\top(k)\, K_i(k) \right\|
$$

with $M_i(k)$ as in (11), (22)can be rewritten as

$$
\Delta e(k) \leq -\gamma_i e(k) \tag{23}
$$

■

Lemma 3. The estimation weight error (9) can be written as:

$$
\begin{aligned}
\widetilde{w}_i^\top(k) &= \left[\epsilon_i^{'}(k) - L_i e(k) - C_i^+ e(k) - C_i^+ \Delta e(k)\right]\\
&\quad \times z_i^+(x(k), u(k))
\end{aligned}
$$

with C_i^+ the ith row of C^+ and $\epsilon_i^{'}(k) = \epsilon_i + d_i(k)$.

Proof: From $(15) - (17)$, we have

$$
\begin{aligned}
e(k) &= y(k) - \widehat{y}(k) = C(x(k) - \widehat{x}(k)) && (24)\\
e(k+1) &= C(x(k+1) - \widehat{x}(k+1))\\
&= C\left(w^{*\top} z(x(k), u(k)) + \epsilon + d(k)\right)\\
&\quad -C\left(w_i^\top(k)\, z_i(\widehat{x}(k), u(k)) + Le(k)\right)\\
&= e(k) + \Delta e(k)
\end{aligned}
$$

then

$$
\begin{aligned}
\widetilde{w}^\top(k)\, z(x(k), u(k)) &= \epsilon + d(k) - Le(k)\\
&\quad -C^+(e(k) + \Delta e(k))
\end{aligned}
$$

and

$$
\begin{aligned}
\widetilde{w}^\top(k) &= \left[\epsilon + d(k) - Le(k) - C^+ e(k) - C^+ \Delta e(k)\right]\\
&\quad \times z^+(x(k), u(k))
\end{aligned}
$$

Then, for the ith element, we have

$$\begin{aligned}\widetilde{w}_i^\top(k) &= \left[\epsilon_i^{'}(k) - L_i e(k) - C_i^+ e(k) - C_i^+ \Delta e(k)\right] \\ &\quad \times z_i^+(x(k), u(k)) \qquad (25)\end{aligned}$$

■

Considering $(10) - (17)$, we establish the main result in the following theorem.

Theorem 2: For the system (15), the RHONO (16) trained with the EKF-based algorithm (10), ensures that the output error (17) and the estimation error (18) are semiglobally uniformly ultimately bounded (SGUUB); moreover, the RHONO weights remain bounded.

Proof: Consider the Lyapunov function candidate, for $e(k)$, $w(k)$, defined as

$$V_i(k) = e^\top(k)\, e(k) + \widetilde{w}_i^\top(k)\, \widetilde{w}_i(k) \qquad (26)$$

whose first difference is:

$$\begin{aligned}\Delta V_i(k) &= V_i(k+1) - V_i(k) \\ &= e^\top(k+1)\, e(k+1) + \widetilde{w}_i^\top(k+1)\, \widetilde{w}_i(k+1) \\ &\quad - e^\top(k)\, e(k) - \widetilde{w}_i^\top(k)\, \widetilde{w}_i(k) \qquad (27)\end{aligned}$$

From (9)and (10), then

$$\widetilde{w}_i(k+1) = \widetilde{w}_i(k) + \eta_i K_i(k)\, e(k) \qquad (28)$$

Let us define

$$\begin{aligned}&\left[\widetilde{w}_i(k) + \eta_i K_i(k)\, e(k)\right]^\top \\ &\times \left[\widetilde{w}_i(k) + \eta_i K_i(k)\, e(k)\right] \\ = \;& \widetilde{w}_i^\top(k)\, \widetilde{w}_i(k) + 2\eta_i \widetilde{w}_i^\top(k)\, K_i(k)\, e(k) \\ &+ \left(\eta_i K_i(k)\, e(k)\right)^\top \left(\eta_i K_i(k)\, e(k)\right) \qquad (29)\end{aligned}$$

From(17) ,then

$$\begin{aligned}e(k+1) &= e(k) + \Delta e(k) \\ e^\top(k+1)\, e(k+1) &= e^\top(k)\, e(k) + e^\top(k)\, \Delta e(k) \\ &\quad + \Delta e^\top(k)\, e(k) + \Delta e^\top(k)\, \Delta e(k) \\ e^\top(k+1)\, e(k+1) - e^\top(k)\, e(k) &= e^\top(k)\, \Delta e(k) \\ &\quad + \Delta e^\top(k)\, e(k) + \Delta e^\top(k)\, \Delta e(k) \qquad (30)\end{aligned}$$

Using (29), (30) and Lemma 3 in (27):

$$\begin{aligned}\Delta V_i(k) &= e^\top(k)\, \Delta e(k) + \Delta e^\top(k)\, e(k) \qquad (31) \\ &\quad + \Delta e^\top(k)\, \Delta e(k) \\ &\quad + 2\eta_i \left[\epsilon_i - L_i e(k) - C_i^+ e(k) - C_i^+ \Delta e(k)\right] \\ &\quad \times z_i^+(k))\, K_i(k)\, e(k) \\ &\quad + \left(\eta_i K_i(k)\, e(k)\right)^\top \eta_i K_i(k)\, e(k)\end{aligned}$$

From Lemma 2, substituting (23) , in (31), then

$$\begin{aligned}\Delta V_i(k) &\leq -2\gamma_i e^\top(k)\, e(k) + \gamma_i^2 e^\top(k)\, e(k) \\ &\quad + 2\eta_i \left[\epsilon_i - L_i e(k) - C_i^+ e(k) + \eta_i \gamma_i C_i^+ e(k)\right] \\ &\quad \times z_i^+(k))\, K_i(k)\, e(k) \\ &\quad + \left(\eta_i K_i(k)\, e(k)\right)^\top \eta_i K_i(k)\, e(k) \qquad (32)\end{aligned}$$

Defining $\delta_i = \eta_i \gamma_i C_i^+ - L_i - C_i^+$, (32) can be written as

$$\begin{aligned}\Delta V_i(k) &\leq -2\gamma_i e^\top(k)\, e(k) + \gamma_i^2 e^\top(k)\, e(k) \\ &\quad + 2\eta_i \left[\epsilon_i + \delta_i e(k)\right] z_i^+(k))\, K_i(k)\, e(k) \\ &\quad + \left(\eta_i K_i(k)\, e(k)\right)^\top \eta_i K_i(k)\, e(k) \\ &\leq -2\gamma_i e^\top(k)\, e(k) + \gamma_i^2 e^\top(k)\, e(k) \\ &\quad + 2\eta_i \epsilon_i z_i^+(k))\, K_i(k)\, e(k) \\ &\quad + 2\eta_i \delta_i e(k)\, z_i^+(k))\, K_i(k)\, e(k) \\ &\quad + \left(\eta_i K_i(k)\, e(k)\right)^\top \eta_i K_i(k)\, e(k) \\ &\leq -2\gamma_i \left\|e(k)\right\|^2 + \gamma_i^2 \left\|e(k)\right\|^2 \\ &\quad + 2\left|\eta_i \epsilon_i z_i^+(k))\, K_i(k)\right| \left\|e(k)\right\| \\ &\quad + 2\left|\eta_i \delta_i z_i^+(k))\, K_i(k)\right| \left\|e(k)\right\|^2 \\ &\quad + \left\|\eta_i K_i(k)\right\|^2 \left\|e(k)\right\|^2\end{aligned}$$

then, there exits $\eta_i > 0$ and L_i such that

$$\Delta V_i(k) < 0, \quad \text{for} \quad \left\|e(k)\right\| > \kappa_i \qquad (33)$$

with

$$\kappa_i = \frac{\left|2\eta_i \epsilon_i \left\|z_i^+ K\right\|\right|}{\left|2\gamma_i - \gamma_i^2 - 2\eta_i \left\|\delta_i z_i^+ K_i\right\| - \eta_i^2 \left\|K_i\right\|^2\right|}$$

From (33) it follows the boundness of $V_i(k)$ for a $k \geq k_T$, that leads the SGUUB of $e(k)$ and $\widetilde{w}_i(k)$. Considering (18) and (24) it is easy too see that the estimation error has an algebraic relation with $e(k)$ for that reason if $e(k)$ is bounded $\widetilde{x}(k)$ is bounded too.

$$\begin{aligned}\widetilde{x}(k) &= C^+ e(k) \\ \left\|\widetilde{x}(k)\right\| &\leq \left\|C^+\right\| \left\|e(k)\right\|\end{aligned}$$

Using (25) and (33) , it is easy too see that the estimation weight error (9) has an algebraic relation with $e(k)$ and $z_i^+(x(k), u(k))$. For that reason if $e(k)$ is bounded and given that $z_i^+(x(k), u(k))$, as defined in (5), is bounded, then $\widetilde{w}(k)$ is bounded too.

$$\begin{aligned}\widetilde{w}_i^\top(k) &= \left[\epsilon_i^{'}(k) - L_i e(k) - C_i^+ e(k) - C_i^+ \Delta e(k)\right] \\ &\quad \times z_i^+(x(k), u(k)) \\ \widetilde{w}_i^\top(k) &\leq \left|\epsilon_i^{'}(k) - L_i e(k) - C_i^+ e(k) - C_i^+ \Delta e(k)\right| \\ &\quad \times \left\|z_i^+(x(k), u(k))\right\|\end{aligned}$$

■

V. Simulation results

In this section, the neural observer is applied to a modified Van der Pol oscillator, whose nonlinear dynamics is represented by the following equation [16]:

$$\begin{aligned} x_1(k+1) &= x_1(k) + Tx_2(k) + d_1(k) \\ x_2(k+1) &= x_2(k) + T\left(-\xi\left(x_1^2(k) - 1\right)x_2(k)\right) \\ &\quad +T\left(-x_1(k) + u(k)\right) + d_2(k) \\ y(k) &= x_1(k) \\ d_1(k) &= 0.1\sin(k) \\ d_2(k) &= 0.1\cos(k) \end{aligned} \tag{34}$$

where variables $x \in \Re^2$, $u \in \Re$, and $y \in \Re$ are the state, input, and output of the system, respectively; $d_1(k)$ and $d_2(k)$ are bounded external disturbances; T is the sampling period, which is fixed at $0.1s$ and ξ is a parameter which nominal value is equal to 2.

To estimate the state x_2 we use the RHONO (16) with $n = 2$ trained with the EKF (10).

$$\begin{aligned} \widehat{x}_1(k+1) &= w_{11}(k)S^2(\widehat{x}_1(k)) \\ &\quad +w_{12}(k)S(\widehat{x}_1(k))S(\widehat{x}_2(k)) \\ &\quad +w_{13}(k)S^2(\widehat{x}_2(k)) + w_{14}(k)S^4(\widehat{x}_2(k)) \\ \widehat{x}_2(k+1) &= w_{21}(k)S^2(\widehat{x}_1(k)) + w_{22}(k)S^3(\widehat{x}_2(k)) \\ &\quad +w_{23}(k)S(\widehat{x}_1(k))S(\widehat{x}_2(k)) \\ &\quad +w_{24}(k)S^2(\widehat{x}_2(k)) + w_{25}(k)S^3(u(k)) \\ \widehat{y}(k) &= \widehat{x}_1(k) \\ u(k) &= \cos\left(\frac{2\pi k}{25}\right) \end{aligned} \tag{35}$$

The training is performed on-line, using a parallel configuration as displayed in Fig. 1. All the NN states are initialized in a random way. The covariances matrices are initialized as diagonals, and the nonzero elements are: $P_1(0) = P_2(0) = 10000$; $Q_1(0) = Q_2(0) = 500$ and $R_1(0) = R_2(0) = 10000$, respectively. The simulation results are presented in Fig. 2, and Fig. 3. They display the time evolution of the estimated states $x_1(k)$ and $x_2(k)$, respectively. The Fig.4. shows the estimation errors. Fig. 5 displays the parametric variation for ξ increment, and Fig. 6 shows the bounded external disturbances.

VI. Conclusions

A RHONN is used to design a Luenberger-like observer for a class of MIMO discrete-time nonlinear system. The RHONO proposed is trained with an EKF-based algorithm. The training of the RHONO is performed on-line in a parallel configuration. The boundness of the output and estimation errors is established on the basis of the Lyapunov approach. Simulation results shows the effectiveness of the proposed RHONO. The results presented in this paper seems important due to

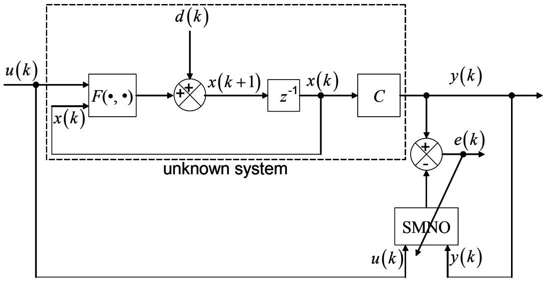

Fig. 1. Observation scheme

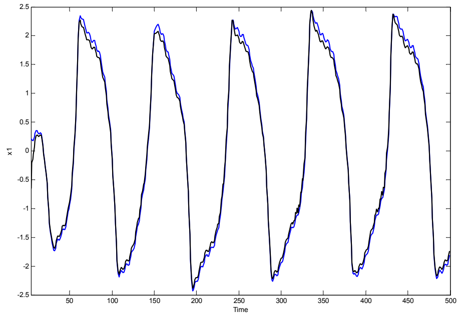

Fig. 2. Time evolution of the state $x_1(k)$ (solid line) and its estimated $\widehat{x}_1(k)$ (dashed line)

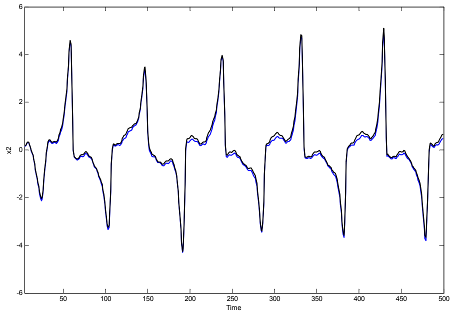

Fig. 3. Time evolution of the state $x_2(k)$ (solid line) and its estimated $\widehat{x}_2(k)$ (dashed line)

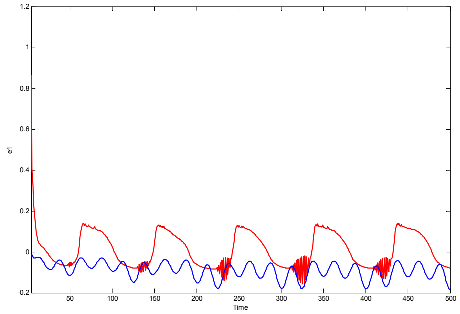

Fig. 4. Etimation errors $\widetilde{x}_1(k)$ (dashed line) and $\widetilde{x}_2(k)$ (solid line).

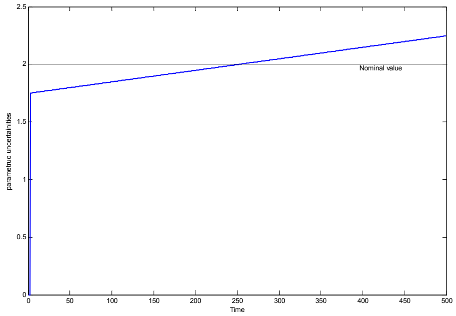

Fig. 5. Uncertainities in parameter ξ

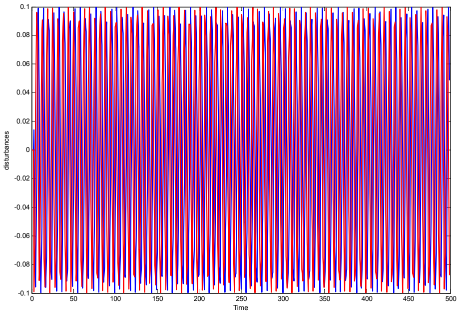

Fig. 6. Disturbances $d_1\,(k)$ (solid line) and $d_2\,(k)$ (dashed line)

the need of observers for unknown or partially unknown nonlinear systems in discrete-time..

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