Design of Plant-Friendly Signals for "Fast" Identification Using Constrained Minimum Crest Factor Multisine Inputs

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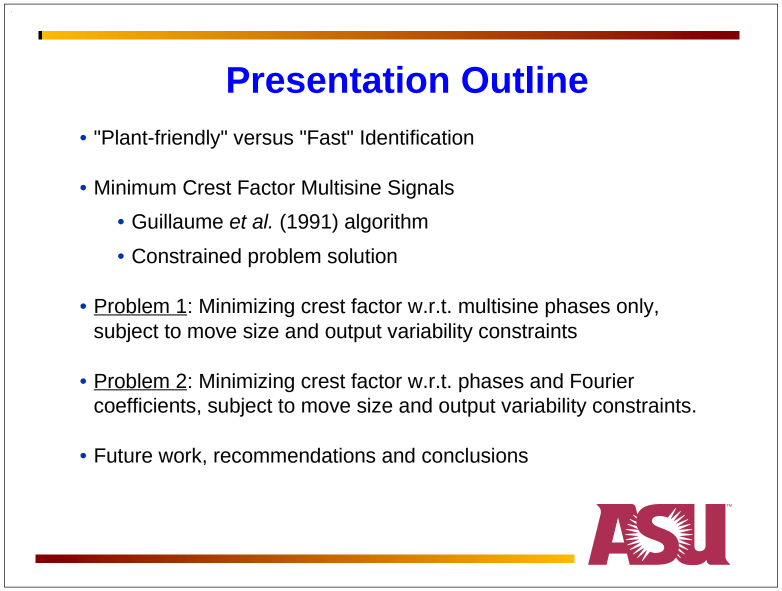
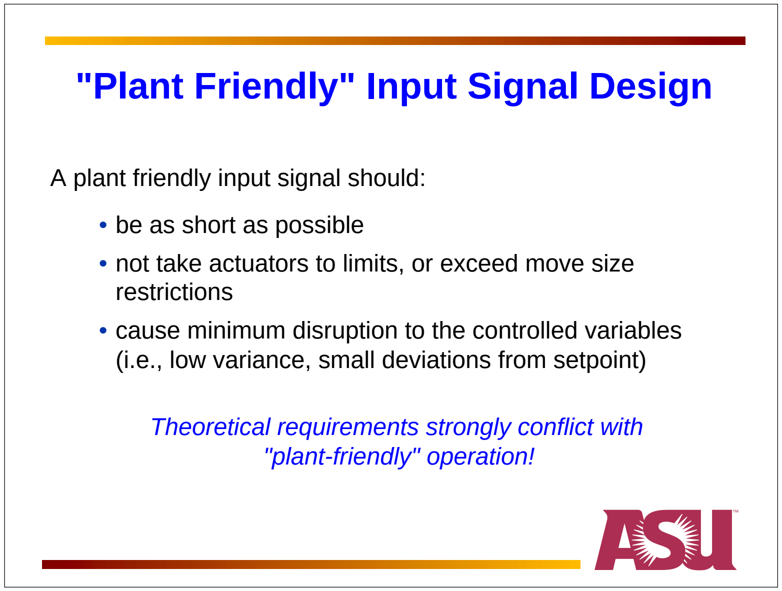


Presentation Outline

- "Plant-friendly" versus "Fast" Identification
- Minimum Crest Factor Multisine Signals
  - Guillaume *et al.* (1991) algorithm
  - Constrained problem solution
- <u>Problem 1</u>: Minimizing crest factor w.r.t. multisine phases only, subject to move size and output variability constraints
- <u>Problem 2</u>: Minimizing crest factor w.r.t. phases and Fourier coefficients, subject to move size and output variability constraints.
- Future work, recommendations and conclusions

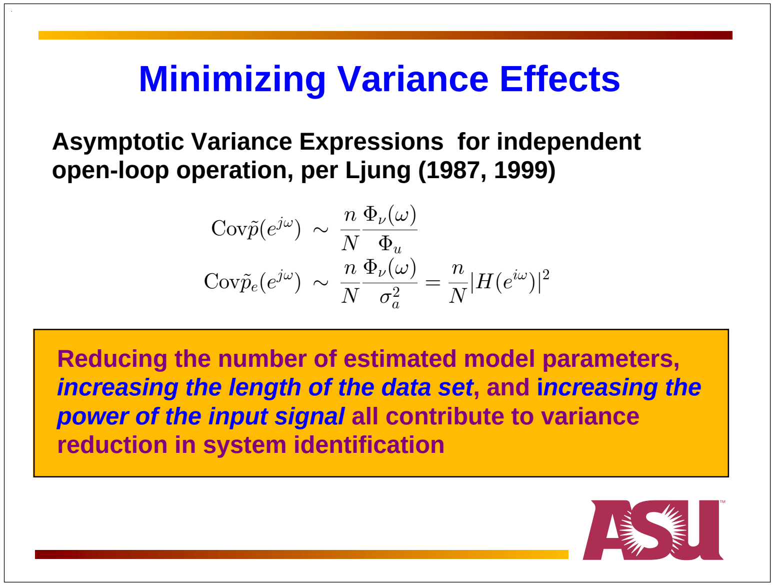


"Plant Friendly" Input Signal Design

A plant friendly input signal should:

- be as short as possible
- not take actuators to limits, or exceed move size restrictions
- cause minimum disruption to the controlled variables (i.e., low variance, small deviations from setpoint)

Theoretical requirements strongly conflict with "plant-friendly" operation!



Minimizing Variance Effects

Asymptotic Variance Expressions for independent open-loop operation, per Ljung (1987, 1999)

$$\mathrm{Cov}\tilde{p}(e^{j\omega}) \sim \frac{n}{N}\frac{\Phi_\nu(\omega)}{\Phi_u}$$

$$\mathrm{Cov}\tilde{p}_e(e^{j\omega}) \sim \frac{n}{N}\frac{\Phi_\nu(\omega)}{\sigma_a^2} = \frac{n}{N}|H(e^{i\omega})|^2$$

Reducing the number of estimated model parameters, *increasing the length of the data set*, and *increasing the power of the input signal* all contribute to variance reduction in system identification

Research Goal

- Develop a design methodology for multisine inputs that incorporates *both* time-domain "plant-friendliness" constraints and frequency-domain information-theoretic requirements.
- Information-theoretic requirements include such considerations as:
  - persistence of excitation
  - independence / orthogonality
  - control-relevance
  - harmonic suppression (meaningful for nonlinear id)

Multsine Input Signals

A multisine input is a deterministic, periodic signal composed of a harmonically related sum of sinusoids,

$$u_s(k) = \lambda \sum_{i=1}^{n_s} \sqrt{2\alpha_i} \cos(\omega_i k T + \phi_i)$$

$T \equiv$ sampling time

$N_s \equiv$ sequence length

$n_s \equiv$ no. of sinusoids, $n_s \leq N_s/2$

$\phi_i \equiv$ phase angle for harmonic i

$\alpha_i \equiv$ relative power $\left(\sum_{i=1}^{n_s} \alpha_i = 1\right)$

$\omega_i = 2\pi i / N_s T$

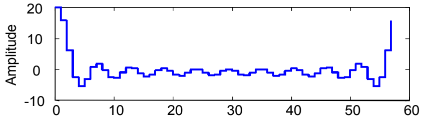
$\lambda \equiv$ scaling factor to insure signal meets $\pm u_{sat}$

The choice of phases ϕ_i, $i = 1, \cdots, n_s$ strongly influences the time-domain realization of the signal.

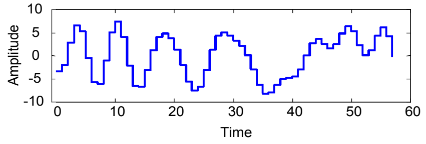
The Crest Factor (CF) of a signal u is the ratio of the ℓ_∞ (or Chebyshev) norm and the ℓ_2-norm

$$CF(u) = \frac{\ell_\infty(u)}{\ell_2(u)}$$

All $\phi_i = 0$, cf $= 4.4721$



ϕ_i selected by Schroeder phase eqn., cf $= 1.8767$

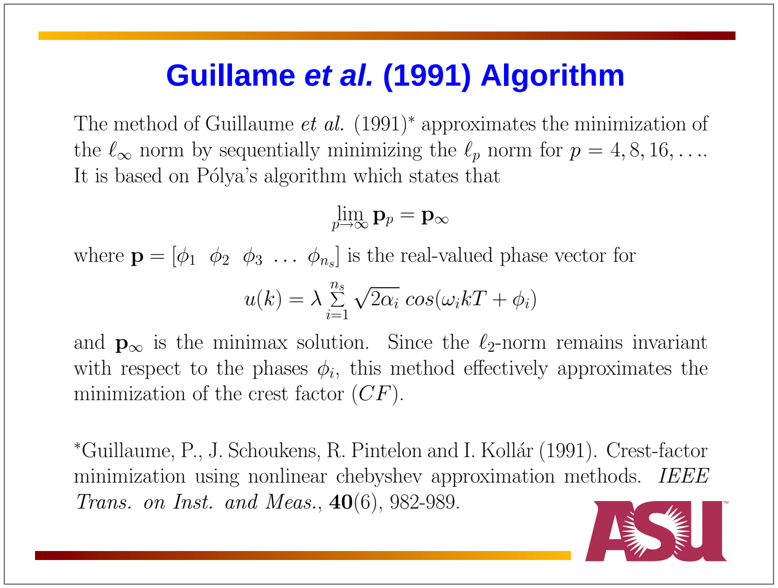


Schroeder, M.R. (*IEEE Trans. Information Theory*, **16**, 85, Jan. 1970)

Problems with Crest Factor Minimization

- Nonlinear optimization problem
- Nonsmooth (as a consequence of the $\mathcal{L}$-infinity norm)
- Nonconvex objective function

Problems are worsened as a result of increasing problem dimension



Guillame *et al.* (1991) Algorithm

The method of Guillaume *et al.* (1991)* approximates the minimization of the ℓ_∞ norm by sequentially minimizing the ℓ_p norm for $p = 4, 8, 16, \ldots$. It is based on Pólya's algorithm which states that

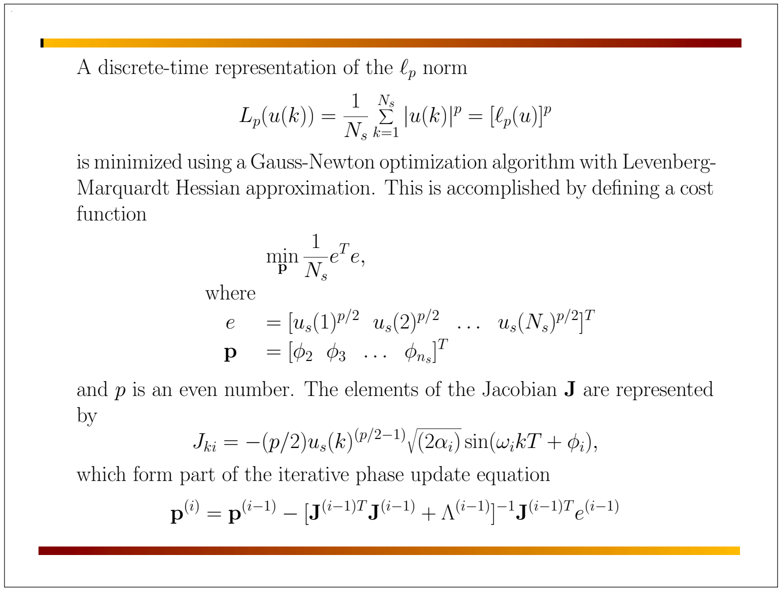
$$\lim_{p\to\infty} \mathbf{p}_p = \mathbf{p}_\infty$$

where $\mathbf{p} = [\phi_1\ \ \phi_2\ \ \phi_3\ \ \ldots\ \ \phi_{n_s}]$ is the real-valued phase vector for

$$u(k) = \lambda \sum_{i=1}^{n_s} \sqrt{2\alpha_i}\, cos(\omega_i kT + \phi_i)$$

and $\mathbf{p}_\infty$ is the minimax solution. Since the ℓ_2-norm remains invariant with respect to the phases ϕ_i, this method effectively approximates the minimization of the crest factor (CF).

*Guillaume, P., J. Schoukens, R. Pintelon and I. Kollár (1991). Crest-factor minimization using nonlinear chebyshev approximation methods. *IEEE Trans. on Inst. and Meas.*, **40**(6), 982-989.



A discrete-time representation of the ℓ_p norm

$$L_p(u(k)) = \frac{1}{N_s}\sum_{k=1}^{N_s} |u(k)|^p = [\ell_p(u)]^p$$

is minimized using a Gauss-Newton optimization algorithm with Levenberg-Marquardt Hessian approximation. This is accomplished by defining a cost function

$$\min_{\mathbf{p}} \frac{1}{N_s} e^T e,$$

where

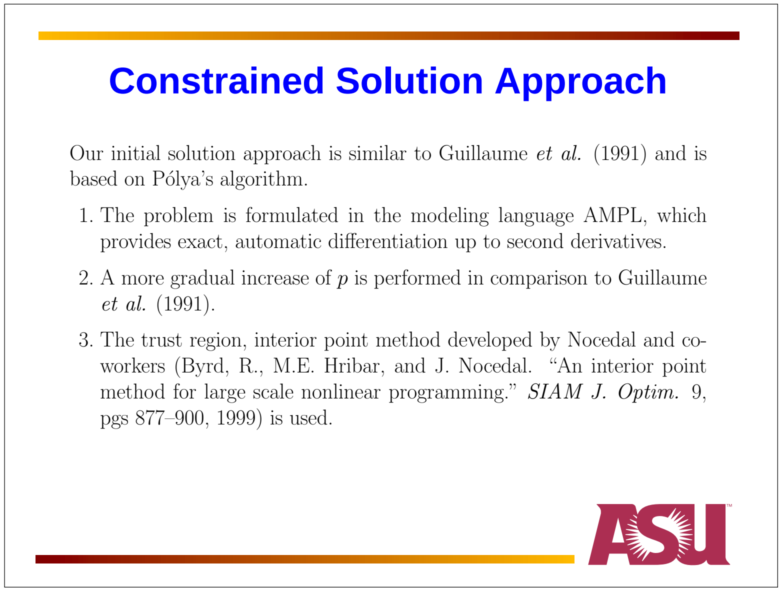
$$\begin{aligned} e &= [u_s(1)^{p/2}\ \ u_s(2)^{p/2}\ \ \ldots\ \ u_s(N_s)^{p/2}]^T \\ \mathbf{p} &= [\phi_2\ \ \phi_3\ \ \ldots\ \ \phi_{n_s}]^T \end{aligned}$$

and p is an even number. The elements of the Jacobian $\mathbf{J}$ are represented by

$$J_{ki} = -(p/2)u_s(k)^{(p/2-1)}\sqrt{(2\alpha_i)}\sin(\omega_i kT + \phi_i),$$

which form part of the iterative phase update equation

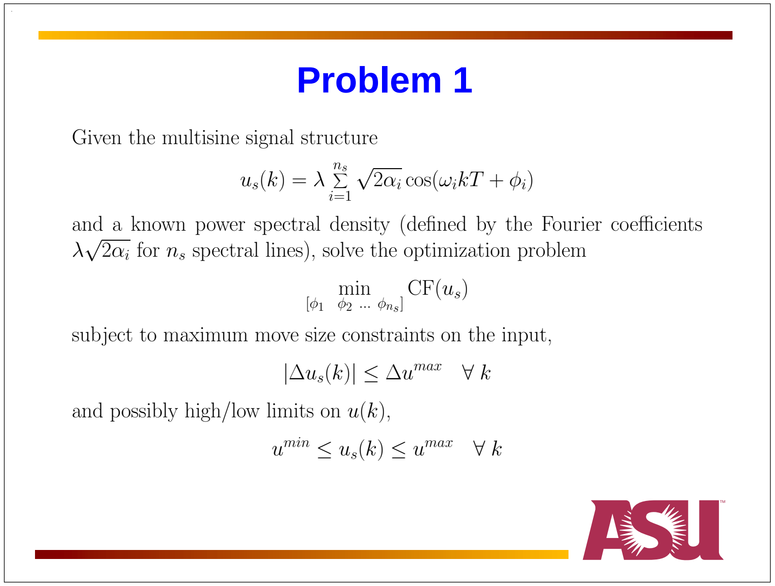
$$\mathbf{p}^{(i)} = \mathbf{p}^{(i-1)} - [\mathbf{J}^{(i-1)T}\mathbf{J}^{(i-1)} + \Lambda^{(i-1)}]^{-1}\mathbf{J}^{(i-1)T}e^{(i-1)}$$



Constrained Solution Approach

Our initial solution approach is similar to Guillaume *et al.* (1991) and is based on Pólya's algorithm.

1. The problem is formulated in the modeling language AMPL, which provides exact, automatic differentiation up to second derivatives.
2. A more gradual increase of p is performed in comparison to Guillaume *et al.* (1991).
3. The trust region, interior point method developed by Nocedal and co-workers (Byrd, R., M.E. Hribar, and J. Nocedal. "An interior point method for large scale nonlinear programming." *SIAM J. Optim.* 9, pgs 877–900, 1999) is used.



Problem 1

Given the multisine signal structure

$$u_s(k) = \lambda \sum_{i=1}^{n_s} \sqrt{2\alpha_i}\cos(\omega_i kT + \phi_i)$$

and a known power spectral density (defined by the Fourier coefficients $\lambda\sqrt{2\alpha_i}$ for n_s spectral lines), solve the optimization problem

$$\min_{[\phi_1\ \ \phi_2\ \ldots\ \phi_{n_s}]} \mathrm{CF}(u_s)$$

subject to maximum move size constraints on the input,

$$|\Delta u_s(k)| \leq \Delta u^{max} \quad \forall\ k$$

and possibly high/low limits on $u(k)$,

$$u^{min} \leq u_s(k) \leq u^{max} \quad \forall\ k$$

Consider a first-order with deadtime plant model

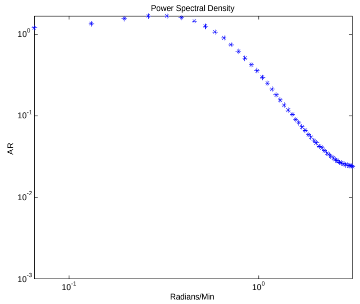
$$\frac{y(s)}{u(s)} = p(s) = \frac{Ke^{-\theta s}}{\tau s+1} = \frac{e^{-3s}}{3s+1}, \quad T = 1 \text{ min}$$

The power spectrum for the input is chosen based on the control-relevant prefilter result presented in Rivera *et al.* 1992*:

$$L(z) = \tilde{p}_e(z)\tilde{p}^{-1}(z)\tilde{\epsilon}(z)\tilde{\eta}(z)(r(z)-d(z))$$

$$\begin{aligned}
\tilde{p}(z) &\equiv \text{plant model} \\
\tilde{p}_e(z) &\equiv \text{noise model} \\
\tilde{\eta}(z) &= \tilde{p}c(1+\tilde{p}c)^{-1} \text{ complementary sensitivity} \\
\tilde{\epsilon}(z) &= (1+\tilde{p}c)^{-1} \text{ sensitivity} \\
r(z)-d(z) &\equiv \text{setpoint/disturbance direction}
\end{aligned}$$

*Rivera, D.E., J.F. Pollard, and C.E. García, (1992) "Control-Relevant Prefiltering: A Systematic Design Approach and Case Study," *IEEE Trans. Autom. Cntrl.*, **37**, 964-974.



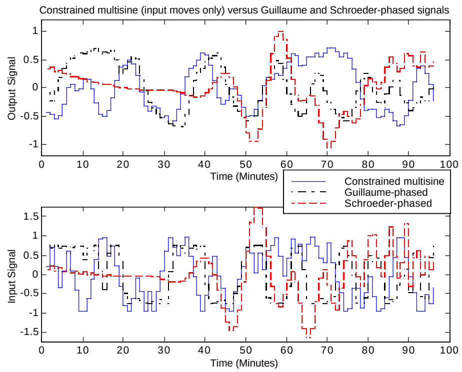
$$u(z) = (z-\alpha)(1-z^{-nk}f(z))z^{-1}f(z)(r(z)-d(z)), \qquad f(z) = \frac{(1-\delta)^2z^2}{(z-\delta)^2}$$

$\delta = e^{-1.555T/\tau_{cl}}$; $\tau_{cl} = 2.25$ min is the desired closed-loop time constant. $\alpha = e^{-T/\tau_{dom}}$; $\tau_{dom} = 4.5$ min is a dominant time constant estimate

Problem 1 Example Results

Signal	CF(u)	max(Δu)	min(u)	max(u)	CF(y)	max(Δy)	min(y)	max(y)
Schroeder-phased	2.7966	1.6214	-1.6384	1.75	2.5892	0.4846	-1.0524	0.9984
Guillaume-phased	1.2173	1.1013	-0.7618	0.7612	1.7020	0.3506	-0.6918	0.6835
min CF(u) Constrained $(\Delta u^{max} = 0.52)$	1.5388	0.5189	-0.9606	0.9628	1.7635	0.3348	-0.6693	0.7168
min CF(y) Unconstrained	2.8144	1.5726	-1.5191	1.7611	1.2068	0.5745	-0.4851	0.4905
min CF(y) $(\Delta u^{max} = 0.6;$ $\Delta y^{max} = 0.45)$	2.1826	0.5156	-1.3571	1.3656	1.9633	0.3371	-0.7730	0.7980

- Maximum move size reduced by more than 50% with only a relatively low (~25% increase) in crest factor



Constrained multisine input signal (with $\Delta u^{max} = 0.52$) compared to Guillaume-phased and Schroeder-phased signals.

Problem 1(b)

Minimize the crest factor of the plant *output*

$$\min_{[\phi_1 \ \ \phi_2 \ \dots \ \phi_{n_s}]} \mathrm{CF}(y)$$

subject to maximum move size constraints on the input

$$|\Delta u_s(k)| \leq \Delta u^{max} \quad \forall \ k$$

output variability constraints

$$|\Delta y(k)| \leq \Delta y^{max} \quad \forall \ k$$

and high/low limits on $u_s(k)$ and $y(k)$

$$u^{min} \leq u_s(k) \leq u^{max} \quad \forall \ k$$

$$y^{min} \leq y(k) \leq y^{max} \quad \forall \ k$$

This scenario assumes that some *a priori* knowledge of the model is available to the optimizer to generate predicted outputs.

Problem 1 Example Results

Signal	$\mathrm{CF}(u)$	$\max(\Delta u)$	$\min(u)$	$\max(u)$	$\mathrm{CF}(y)$	$\max(\Delta y)$	$\min(y)$	$\max(y)$
Schroeder-phased	2.7966	1.6214	-1.6384	1.75	2.5892	0.4846	-1.0524	0.9984
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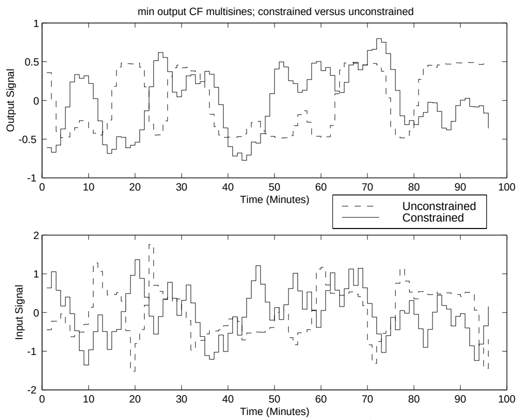
min $\mathrm{CF}(y)$ ($\Delta u^{max} = 0.6$; $\Delta y^{max} = 0.45$)	2.1826	0.5156	-1.3571	1.3656	1.9633	0.3371	-0.7730	0.7980

Constrained problem solution sharply reduces the input move size and output changes. There is an unavoidable increase in the output CF, but the input CF and input/output spans are brought within acceptable levels.



Minimum *output* crest factor multisine inputs (constrained and unconstrained).

Problem 2

Given the multisine signal structure below and a known power spectrum (defined by the Fourier coefficients $\lambda\sqrt{2\alpha_i}$ for n_s spectral lines)

$$u_s(k) = \lambda \sum_{i=1}^{n_s} \sqrt{2\alpha_i} \cos(\omega_i kT + \phi_i) + \sum_{i=n_s+1}^{n_a+n_s} \hat{a}_i \cos(\omega_i kT + \phi_i^a)$$

where $n_a + n_s \leq N_s/2$ and $\hat{a}_i$ and ϕ_i^a represent the Fourier coefficients and phases, respectively, for n_a spectral lines beyond those defined in the user-specified spectrum, solve the optimization problem

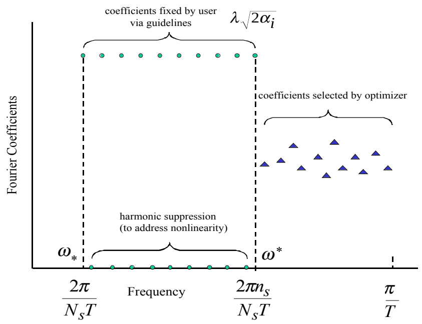
$$\min_{[\phi_1 \ \ \phi_2 \ \dots \ \phi_{n_s}],[\phi^a_{n_s+1} \ \ \phi^a_{n_s+2} \ \dots \ \phi^a_{n_s+n_a}],[\hat{a}_{n_s+1} \ \ \hat{a}_{n_s+2} \ \dots \ \hat{a}_{n_s+n_a}]} \mathrm{CF}(u_s)$$

subject to maximum move size constraints on the input,

$$|\Delta u_s(k)| \leq \Delta u^{max} \quad \forall \ k$$

and possibly high/low limits on $u(k)$,

$$u^{min} \leq u_s(k) \leq u^{max} \quad \forall \ k$$

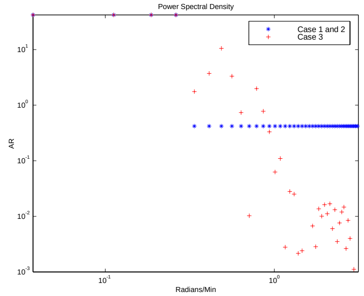


Flat spectrum displaying harmonic suppression with optimized Fourier coefficients. The known part of the spectrum is defined by:

$$\omega_* = \frac{1}{\beta_s \tau_{dom}^H} \leq \omega \leq \frac{\alpha_s}{\tau_{dom}^L} = \omega^*.$$

τ_{dom}^H and τ_{dom}^L correspond to the high and low estimates of the dominant time constant of the system. α_s and β_s correspond to user-decisions on the high and low frequency content of the signal, respectively.

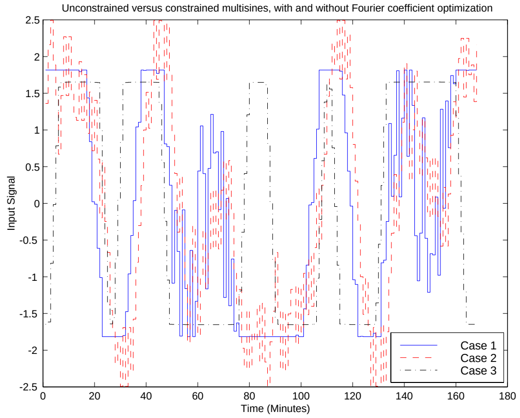
Problem 2 Example Results



Signal	CF(u)	max(Δu)	min(u)	max(u)
Case 1	1.2274	2.2618	-1.8164	1.8164
Case 2	1.6854	0.8	-2.4942	2.4942
Case 3	1.0947	0.8	-1.6532	1.6532

Problem 2 example results; Case 1 (unconstrained), Case 2 ($\Delta u^{max} = 0.8$), Case 3 ($\Delta u^{max} = 0.8$; high-frequency Fourier coefficients optimized).

Problem 2 Example Results (Cont.)



Problem 2 example results; Case 1 (unconstrained), Case 2 ($\Delta u^{max} = 0.8$), Case 3 ($\Delta u^{max} = 0.8$; high-frequency Fourier coefficients optimized).

Summary, Conclusions & Future Results

- A *constrained* problem formulation allowing the user to specify desirable frequency-domain and time-domain specifications in low crest factor multisines has been presented.
- An initial yet powerful numerical solution has been developed and its usefulness has been demonstrated in problems arising from control-relevant and nonlinear system identification.
- Future work includes:
  - multivariable problem formulations
  - "direct" minimization of the L-infinity norm
  - "fast" solution (as in quasi-realtime)