

# Control-Relevant Curvefitting for Plant-Friendly Multivariable System Identification

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**Abstract**—A control-relevant parameter estimation algorithm is developed in this paper for curvefitting Empirical Transfer Function Estimates (ETFEs) with orthogonal (i.e., zippered) frequency grids to discrete-time parametric Matrix Fraction Description models. Such ETFEs arise from DFT analysis of identification data generated from constrained, plant-friendly multisine inputs as developed by the authors' previous work. This curvefitter minimizes model estimation error using pre/post frequency-dependent weighting matrices as functions of the closed-loop dynamics. The control-relevant multivariable parameter estimation procedure is illustrated with an example case study based on the Shell Heavy Oil Fractionator problem.

## I. INTRODUCTION

As an effective tool for model implementation in advanced control systems, control-relevant system identification has been studied extensively by control engineers, e.g., [1], [2], [3]. An integrated system identification and control methodology can provide a more appropriate model for control system design, possibly at the cost of compromising the open-loop fit [3]. Specifically, a control-relevant parameter estimation algorithm is developed in this paper for curvefitting ETFEs with orthogonal (i.e., zippered) frequency grids to discrete-time Matrix Fraction Description (MFD) parametric models. Such ETFEs arise from the DFT analysis of identification data generated from constrained, plant-friendly multisine inputs as previously developed by Braun *et al.* [4] and H. Lee *et al.* [5].

Previous work on a multivariable frequency response curvefitting includes an approach using a scalar polynomial as a common denominator with a scalar frequency-dependent weighting function proposed by Bayard [1]. Rivera and Gaikwad [3] extend Bayard's approach [1] so that a different denominator polynomial can be estimated for each output channel by the use of diagonal matrix

polynomial. They also apply pre/post frequency-dependent weighting matrices to support control relevance. de Callafon *et al.* [2] apply a full-matrix polynomial MFD model that allows the flexibility to specify individual transfer function elements in multivariable systems.

In this paper, we present a frequency-weighted curvefitting algorithm that utilizes a dataset executed in a plant-friendly way for a control-relevant parameter estimation procedure. In particular, a model representation using full-polynomial matrices [2] is implemented in our curvefitting algorithm that supports "zippered" frequency grids with possible harmonic suppression; such signals arise from the multisine inputs per H. Lee *et al.* (2003). A numerical approach based on Sanathanan-Koerner iteration [6] and Gauss-Newton optimization [1] is utilized in the algorithm. A modified system taken from a subset of the Shell Heavy Oil fractionator plant [7] is used as an example study that illustrates how the model estimation error is shifted in frequency-domain for achieving control-relevance by the curvefitter.

This paper is organized as follows: Section 2 describes a formulation of control-relevant parameter estimation problem. Section 3 presents frequency response generation based on plant-friendly multisine input signals. Section 4 details a discrete-time MFD model formulation while Section 5 focuses on a frequency-response curvefitting with pre- and post-weights and the numerical solution for control-relevant parameters. Section 6 describes an example case study and Section 7 presents summary and conclusions.

## II. CONTROL-RELEVANT PARAMETER ESTIMATION

A key feature of control-relevant parameter estimation is that emphasizes closed-loop requirements during the estimation procedure. In other words, the goal is to obtain a model  $\tilde{P}$  representing a system  $P$  that is best suited for the end use of model, which is control system design. To

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this end, the work of Gaikwad and Rivera [3] established that such a parameter estimation problem can be cast as a pre- and post-weighted 2-norm minimization,

$$\min_E \|G_{21}EG_{12}\|_F^2 \approx \min_E \|W_y \tilde{S} E_m \tilde{H}(r-d)\|_2^2 \quad (1)$$

subject to the condition that

$$\sup_\omega \rho(E_m \tilde{H}) < 1, \quad -\pi \leq \omega \leq \pi \quad (2)$$

where the pre- and post-weights are functions of the closed-loop transfer functions as  $\tilde{S} = (I + \tilde{P}C)^{-1}$ ,  $\tilde{H} = \tilde{P}C(I + \tilde{P}C)^{-1}$  and  $E_m = (P - \tilde{P})\tilde{P}^{-1}$ .  $\rho(E_m \tilde{H})$  arises from the Small Gain Theorem and can be used as a sufficient condition for nominal stability. Control-relevant weights defined by  $G_{12}$  and  $G_{21}$  will dictate which frequency bandwidth is the most important for control system design. For a more detailed discussion of the CRPEP formulation the reader is referred to [3].

### III. PLANT-FRIENDLY, ZIPPERED MULTISINE INPUTS

Multisine signals are deterministic, periodic signals whose power spectrum can be directly specified by the user. A conceptual design scheme of a zippered power spectrum multisine input signal is illustrated in Figure 1 and the detailed design guideline is described in [5].

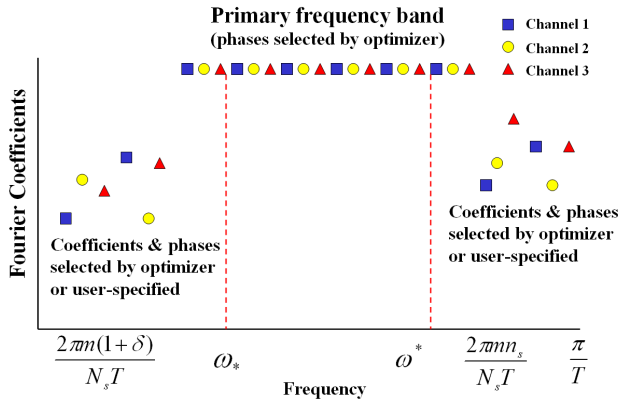


Fig. 1. A conceptual design of a standard “zippered” spectrum for a three-channel signal.

A multisine input  $u_j(k)$  for the  $j$ -th channel of a multi-variable system with  $m$  inputs can be defined as,

$$u_j(k) = \sum_{i=1}^{m\delta} \hat{\delta}_{ji} \cos(\omega_i kT + \phi_{ji}^\delta) + \sum_{i=m\delta+1}^{m(\delta+n_s)} \alpha_{ji} \cos(\omega_i kT + \phi_{ji}) + \sum_{i=m(\delta+n_s)+1}^{m(\delta+n_s+n_a)} \hat{a}_{ji} \cos(\omega_i kT + \phi_{ji}^a), \quad j = 1, \dots, m \quad (3)$$

where  $T$  is sampling time,  $N_s$  is the sequence length,  $m$  is the number of channels,  $\delta$ ,  $n_s$ ,  $n_a$  are the number of sinusoids per channel ( $m(\delta+n_s+n_a) = N_s/2$ ),  $\phi_{ji}^\delta$ ,  $\phi_{ji}$ ,  $\phi_{ji}^a$

are the phase angles,  $\alpha_{ji}$  represents the Fourier coefficients defined by the user,  $\hat{\delta}_{ji}$ ,  $\hat{a}_{ji}$  are the “snow effect” Fourier coefficients [8], and  $\omega_i = 2\pi i/N_s T$  is the frequency grid.

A consistent estimate of the frequency response is obtained from the observed input and output data via an ETFE. The frequency responses are averaged from  $r$ -cycles of input and output data by DFT analysis according to

$$g_{h\ell}(\omega_i) = \frac{\frac{1}{r} \sum_{s=1}^r Y_h^s(\omega_i)}{U_\ell(\omega_i)} \quad (4)$$

$$Y(\omega_i) = DFT(y), \quad U(\omega_i) = DFT(u)$$

where  $g_{h\ell}(\omega_i)$  is computed on frequencies when  $U(\omega_i) \neq 0$ .

### IV. MATRIX FRACTION DESCRIPTION MODEL REPRESENTATION

A frequency-domain identification procedure for multivariable systems using MFD is described in [2] where frequency responses are given as

$$\mathcal{G} := \{G(\omega_i) | G(\omega_i) \in \mathbb{C}^{p \times m}, \text{ for } i \in [1, 2, \dots, N]\} \quad (5)$$

The frequency response data set  $\mathcal{G}$  consists of complex matrices  $G(\omega_i)$ . A model  $\tilde{P}$  of  $m$  inputs and  $p$  outputs is approximated into a linear parametric, real rational transfer function, formed by either a left or right matrix polynomial fractional description.

$$\text{Left MFD } \tilde{P}(\xi^{-1}, \theta) = A(\xi^{-1}, \theta)^{-1} B(\xi^{-1}, \theta) \quad (6)$$

$$\text{Right MFD } \tilde{P}(\xi^{-1}, \theta) = B(\xi^{-1}, \theta) A(\xi^{-1}, \theta)^{-1} \quad (7)$$

where  $\xi(\omega_i) = j\omega_i$  in a continuous time model, whereas  $\xi(\omega_i) = e^{j\omega_i T}$  represents the shift operator in a discrete-time model. For both the left and right MFD, the polynomial matrix  $B$  is defined as a function of  $\xi^{-1}$  and  $\theta$

$$B(\xi^{-1}, \theta) = \sum_{k=d}^{d+b-1} B_k \xi^{-k}, \quad B_k \in \mathbb{R}^{p \times m} \quad (8)$$

where  $d$  denotes the number of leading zero matrix coefficients and  $b$  the number of non-zero matrix coefficients in the  $B(\xi^{-1}, \theta)$ . For the left MFD, the  $A$  polynomial is parameterized by

$$A(\xi^{-1}, \theta) = I_{p \times p} + \xi \sum_{k=1}^a A_k \xi^{-k+1}, \quad A_k \in \mathbb{R}^{p \times p} \quad (9)$$

and for the right MFD

$$A(\xi^{-1}, \theta) = I_{m \times m} + \xi \sum_{k=1}^a A_k \xi^{-k+1}, \quad A_k \in \mathbb{R}^{m \times m} \quad (10)$$

where  $a$  denotes the number of non-zero coefficients in the polynomial  $A(\xi^{-1}, \theta)$ . The parameter  $\theta$  consists of the corresponding unknown matrix coefficients in the  $A$  and

$B$  polynomials. In this paper, we utilize the left MFD parameterization for control-relevant parameter estimation purposes.

The structural parameters  $d_{i,j}$ ,  $b_{i,j}$  and  $a_{i,j}$  that are specified for each of the elements of the polynomial matrices and  $A$  and  $B$  separately, with

$$d := \min\{d_{i,j}\}, b := \max\{b_{i,j}\}, a := \max\{a_{i,j}\} \quad (11)$$

The model order of polynomial estimation via MFD has a limitation on the McMillan degree of the resulting estimate  $\tilde{P}(\xi, \theta)$ . For a more detailed discussion on the exact relation between the McMillan degree, the row degree of the polynomial matrices  $A(\xi^{-1}, \theta)$ ,  $B(\xi^{-1}, \theta)$  and the observability indices of a model computed by  $A(\xi^{-1}, \theta)^{-1}B(\xi^{-1}, \theta)$  the reader is referred to [9].

The model error between the process and model is represented as

$$E(\omega_i, \theta) = G(\omega_i) - \tilde{P}(\omega_i) \text{ for } i \in [1, 2, \dots, N] \quad (12)$$

where  $\tilde{P}$  is given by Left-MFD (6) and  $E(\omega_i, \theta)$  is

$$\begin{aligned} E(\omega_i, \theta) &= G(\omega_i) - A(\xi(\omega_i)^{-1}, \theta)^{-1}B(\xi(\omega_i)^{-1}, \theta) \\ &= A(\xi(\omega_i)^{-1}, \theta)^{-1}\tilde{E}(\omega_i) \end{aligned} \quad (13)$$

where  $\tilde{E}(\omega_i) = G(\omega_i) - \theta\Phi(\omega_i)$  and  $\theta$  and  $\Phi$  are given as

$$\theta = [B_d \dots B_{d+b-1} A_1 \dots A_a] \in R^{p \times (mb+pa)} \quad (14)$$

$$\Phi(\omega_i) = \begin{bmatrix} I_{m \times m} \xi(\omega_i)^{-d} \\ \vdots \\ I_{m \times m} \xi(\omega_i)^{-(d+b-1)} \\ G(\omega_i) \xi(\omega_i)^{-1} \\ \vdots \\ G(\omega_i) \xi(\omega_i)^{-a} \end{bmatrix} \quad (15)$$

If zippered multisine signals are applied for simultaneously exciting all the input channels, a permutation matrix,  $T_m$ , should be applied to ensure only the relevant frequencies in the MFD models are evaluated such that

$$\tilde{E}(\omega_i) = (G(\omega_i) - \theta\Phi(\omega_i))T_m(\omega_i) \quad (16)$$

where  $T_m$  is defined by

$$T_m(\omega_i) = \text{diag}(0, \dots, \underbrace{1}_{j_{th}}, \dots, 0), T_m \in \mathfrak{R}^{m \times m} \quad (17)$$

If  $j_{th}$  input channel has non-zero power at  $\omega_i$  based on the zippered frequency grids, only  $T_m^{(j,j)}(\omega_i) = 1$  and all the other elements are zero.  $T_m(\omega_i)$  can also be applied to harmonically-suppressed frequency grids, which is a consideration in the identification of nonlinear systems.

## V. FREQUENCY-WEIGHTED CURVE-FITTING

Non-weighted parameter estimation can be directly accomplished from the minimization of  $\|E(\omega_i, \theta)\|_2^2$ , which is solved by an iterative least squares method, i.e., Sanathanan-Koerner (SK) iteration method [6]. The S-K model provides an initial set of parameters for Gauss-Newton optimization [1]. Incorporating control-relevant weights, the weighted error is represented as

$$\tilde{E}_w(\omega_i, \theta) = \tilde{W}_2(\omega_i, \theta) \tilde{E}(\omega_i, \theta) W_1(\omega_i, \theta) \quad (18)$$

where  $\tilde{W}_2(\omega_i, \theta) = -W_y(\omega_i) \tilde{S}(\omega_i, \theta)A(\omega_i, \theta)^{-1}$  and  $W_1(\omega_i, \theta) = \tilde{P}^{-1}(\omega_i, \theta) \tilde{H}(\omega_i, \theta)(r - d)$ . All the matrices in  $\tilde{E}_w$  are dependent on frequency and parameters. Thus, an iterative procedure is utilized in this problem such as [6]. The  $\theta$  in step  $t$  is obtained by taking the weights,  $W_2(\omega_i, \theta_{t-1})$  and  $W_1(\omega_i, \theta_{t-1})$ , based on the previous model parameter  $\theta_{t-1}$ . This iterative weighted error equation is denoted by

$$\tilde{E}_w(\omega_i, \theta_{t-1}, \theta) = \tilde{W}_2(\omega_i, \theta_{t-1}) \tilde{E}(\omega_i, \theta) W_1(\omega_i, \theta_{t-1}) \quad (19)$$

Now a parameter vector  $\theta_t$  is estimated from the minimization of  $\tilde{E}_w(\omega_i, \theta_{t-1}, \theta)$  from

$$\theta_t = \arg \min_{\theta \in \mathfrak{R}} \sum_{k=1}^N \|\tilde{W}_2(\omega_i, \theta_{t-1}) \tilde{E}(\omega_i, \theta) W_1(\omega_i, \theta_{t-1})\|_2^2 \Delta\omega_i \quad (20)$$

where  $\Delta\omega_i$  represents the frequency interval for zippered or/and harmonic-suppressed input power spectra. However, the computation of minimization of (20) is much more difficult than a non-weighted error formulation because of the *pre/post* weights on  $\tilde{E}_w$ . The Kronecker vector and Kronecker product are applied to the weighted error matrix.

With matrices  $X$ ,  $Y$ , and  $Z$  with appropriate dimensions, the matrix product  $C = XYZ$  can be obtained by using Kronecker vector and Kronecker product such that

$$\text{vec}(C) = [Z^T \otimes X] \text{vec}(Y)$$

Considering matrices  $X \in C^{n_1 \times n_2}$  and  $Y \in C^{m_1 \times m_2}$ , the Kronecker vector  $\text{vec}(X)$  and Kronecker product  $X \otimes Y$  operators are defined by

$$\text{vec}(X) := \begin{bmatrix} x_{1,1} \\ x_{2,1} \\ \vdots \\ x_{n_1, n_2} \end{bmatrix} X \otimes Y := \begin{bmatrix} x_{1,1}Y & \dots & x_{1, n_2}Y \\ \vdots & \dots & \vdots \\ x_{n_1, 1}Y & \dots & x_{n_1, n_2}Y \end{bmatrix} \quad (21)$$

where  $\text{vec}(X) \in C^{n_1 n_2 \times 1}$  and  $X \otimes Y \in C^{n_1 n_2 \times 1}$ .

As the Frobenius-norm is still valid with the Kronecker operator  $\|X\|_F^2 = \|\text{vec}(X)\|_F^2$  for an arbitrary matrix  $X$ , the minimization objective function of  $\tilde{E}_w$  can be rewritten

in terms of the Kronecker operators. Now,  $\tilde{E}_w$  is rewritten as

$$\begin{aligned}\tilde{E}_w &= \tilde{W}_2 (G - \theta \Phi) T_m W_1 \\ &= (\tilde{W}_2 G T_m W_1) - (\tilde{W}_2 \theta \Phi T_m W_1)\end{aligned}\quad (22)$$

and taking Kronecker *vec* operator to  $\tilde{E}_w$

$$\begin{aligned}\text{vec}(\tilde{E}_w) &= \underbrace{\text{vec}(\tilde{W}_2 G T_m W_1)}_{G_\omega} - \underbrace{[(\Phi T_m W_1)^T \otimes \tilde{W}_2]}_{\Phi_\omega} \bar{\theta} \\ &= G_\omega - \Phi_\omega \bar{\theta}\end{aligned}\quad (23)$$

where  $\text{vec}(\theta) = \bar{\theta}$ . As a result, a control-relevant parameter is obtained by

$$\theta_t^{CRPEP} = \arg \min_{\theta \in \mathcal{R}} \|G_\omega - \Phi_\omega \bar{\theta}\|_F^2 \quad (24)$$

where  $G_\omega$  and  $\Phi_\omega$  are formulated as

$$\begin{aligned}G_\omega &:= \begin{bmatrix} \text{vec}(\Re\{\tilde{W}_2(\omega_1, \theta_{t-1})G(\omega_1)T_m(\omega_1)W_1(\omega_1, \theta_{t-1})\Delta\omega_1\}) \\ \vdots \\ \text{vec}(\Re\{\tilde{W}_2(\omega_N, \theta_{t-1})G(\omega_N)T_m(\omega_N)W_1(\omega_N, \theta_{t-1})\Delta\omega_N\}) \\ \text{vec}(\Im\{\tilde{W}_2(\omega_1, \theta_{t-1})G(\omega_1)T_m(\omega_1)W_1(\omega_1, \theta_{t-1})\Delta\omega_1\}) \\ \vdots \\ \text{vec}(\Im\{\tilde{W}_2(\omega_N, \theta_{t-1})G(\omega_N)T_m(\omega_N)W_1(\omega_N, \theta_{t-1})\Delta\omega_N\}) \end{bmatrix} \\ \Phi_\omega &:= \begin{bmatrix} \Re\{[\Phi(\omega_1)T_m(\omega_1)W_1(\omega_1, \theta_{t-1})\Delta\omega_1]^T \otimes \tilde{W}_2(\omega_1, \theta_{t-1})\} \\ \vdots \\ \Re\{[\Phi(\omega_N)T_m(\omega_N)W_1(\omega_N, \theta_{t-1})\Delta\omega_N]^T \otimes \tilde{W}_2(\omega_N, \theta_{t-1})\} \\ \Im\{[\Phi(\omega_1)T_m(\omega_1)W_1(\omega_1, \theta_{t-1})\Delta\omega_1]^T \otimes \tilde{W}_2(\omega_1, \theta_{t-1})\} \\ \vdots \\ \Im\{[\Phi(\omega_N)T_m(\omega_N)W_1(\omega_N, \theta_{t-1})\Delta\omega_N]^T \otimes \tilde{W}_2(\omega_N, \theta_{t-1})\} \end{bmatrix}\end{aligned}\quad (25)$$

The Kronecker operators transform the estimation of large-size models into a convenient matrix structure for seeking a solution of  $\theta$ .

Taking the weighted error function into the one-step least squares method, the parameter  $\bar{\theta}$  is obtained as the minimum of the following function

$$F = \|\text{vec}(\tilde{E}_w)\|_2^2 = (G_\omega - \Phi_\omega \bar{\theta})^T (G_\omega - \Phi_\omega \bar{\theta}) \quad (27)$$

and a solution is found when  $\frac{\partial F}{\partial \bar{\theta}} = 0$

$$\bar{\theta} = (\Phi_\omega^T \Phi_\omega)^{-1} (\Phi_\omega^T G_\omega) \quad (28)$$

As an iterative minimization procedure, the Hessian-Newton matrix is applied in this problem since  $F$  is twice differentiable with respect to  $\bar{\theta}$  and the iterative procedure is given

$$\bar{\theta}_{k+1} = \bar{\theta}_k - [2\Phi_\omega^T \Phi_\omega]^{-1} [-2\Phi_\omega^T (G_\omega - \Phi_\omega \bar{\theta}_k)] \quad (29)$$

where  $H(\theta_k) = 2\Phi_\omega^T \Phi_\omega$ . For numerical convergence in the iteration, a set of termination criteria can be given as follows

$$\left| \frac{\bar{\theta}_{k+1} - \bar{\theta}_k}{\bar{\theta}_{k+1}} \right| \leq \epsilon_1 \quad (30)$$

$$\left| \frac{\|\tilde{W}_2 \tilde{E} W_1\|_{2, (k+1)}^2 - \|\tilde{W}_2 \tilde{E} W_1\|_{2, (k)}^2}{\|\tilde{W}_2 \tilde{E} W_1\|_{2, (k+1)}^2} \right| \leq \epsilon_2 \quad (31)$$

If the two criteria are satisfied simultaneously, we can terminate the iteration loop at step  $k+1$  with user specifications of  $\epsilon_1$  and  $\epsilon_2$ . A flowchart of this control-relevant parameter estimation is illustrated in Figure 2.

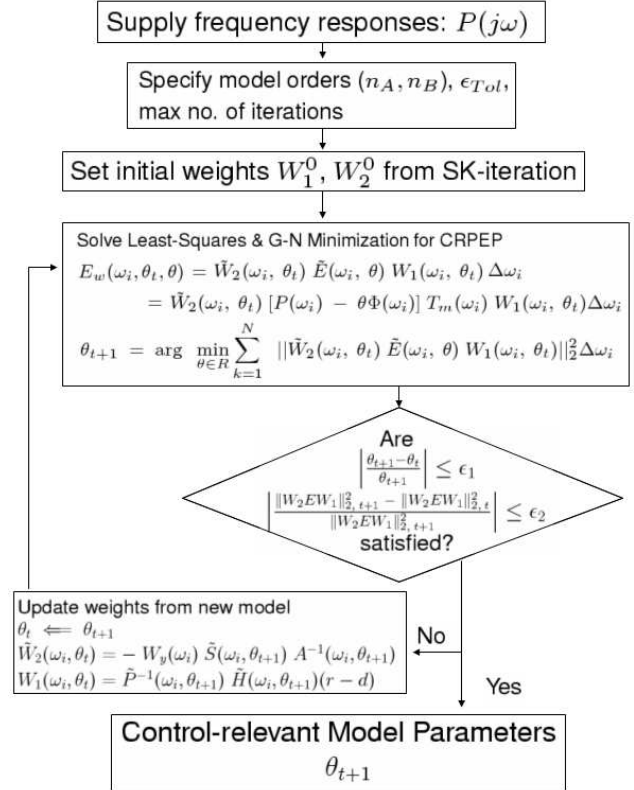


Fig. 2. Flowchart for Solution of the CRPEP

## VI. EXAMPLE OF CONTROL-RELEVANT PARAMETER ESTIMATION TO SHELL HEAVY OIL FRACTIONATOR

A  $2 \times 2$  multivariable system consisting of a subset of the Shell Heavy-Oil Fractionator Plant [7] is taken as an example case study for control-relevant parameter estimation with MPC. The process is a linear and has time-delay in each transfer function element. A modified model from the Shell Heavy-Oil Fractionator is obtained by increasing the time-delay as

$$\begin{aligned}y(t) &= \begin{bmatrix} \frac{4.05}{50s+1} \exp^{-50s} & \frac{1.77}{60s+1} \exp^{-60s} \\ \frac{5.39}{50s+1} \exp^{-50s} & \frac{5.72}{60s+1} \exp^{-60s} \end{bmatrix} u(t) \\ &+ \begin{bmatrix} \frac{1.44}{40s+1} \exp^{-27s} \\ \frac{1.83}{20s+1} \exp^{-15s} \end{bmatrix} d(t)\end{aligned}\quad (32)$$

where the physical outputs and inputs are

$$y(t) = \begin{bmatrix} \text{Top End. Product} \\ \text{Side End. Product} \end{bmatrix} \quad (33)$$

$$u(t) = \begin{bmatrix} \text{Top Draw} \\ \text{Side Draw} \end{bmatrix} d(t) = [\text{URDuty}] \quad (34)$$

Increasing in time delay creates the opportunity for large bias in model estimation which will contrast the weighted vs. unweighted approaches.

### A. Input Signal Design and Open-loop Experiment

The input signal design parameters are obtained from *a priori* knowledge of the system:  $\tau_{dom}^H = 74$ ,  $\tau_{dom}^L = 48$ , and feasible multisine design variables are obtained as  $T = 4\text{min}$ ,  $n_s = 10$ ,  $N_s = 698$ , and  $hf = 0.0$ . An open-loop identification experiment is performed using zippered multisine input signals(see Figure 3). This dataset is used for calculating ETFE values (Figure 4) and solving the control-relevant parameter estimation problem.

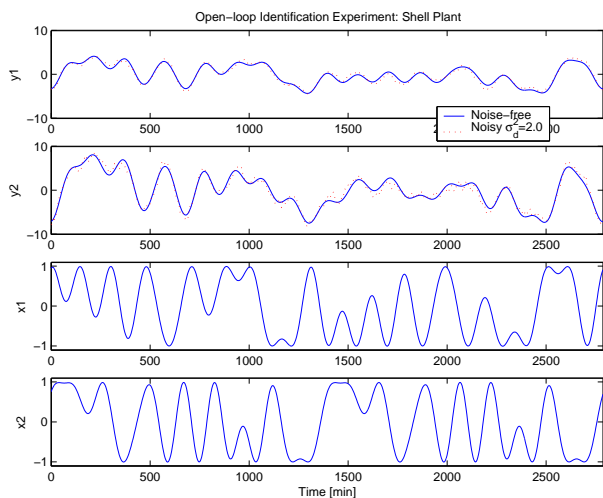


Fig. 3. Open-loop experiment with orthogonal multisine input signals per (32) ( $\sigma_d^2 = 2.0$  for noisy conditions)

### B. Solving control-relevant parameter estimation problem

The results under noise-free conditions are not shown in this paper because of the space limitations. Instead, we will demonstrate the results under noisy conditions in this case study. We utilize ten cycles of data under noisy conditions ( $\sigma_d^2 = 2.0$ ) to obtain frequency responses. These are shown in Figure 3. Tuning parameters for the MPC controller that defines the control-relevant weights are: PH=50, MH=10, Ywt=[1 1], Uwt=[32 40] with setpoint direction [1 -1]. The curvefits, shown in Figure 4, suffer mismatch in most of high frequencies while only the weighted curvefits are close to ETFEs in the low frequencies. Figure 5 displays  $\rho(E_m \tilde{H})$  for both the unweighted and weighted models. The spectral radius is shifted by applying the control-relevant weights such that the weighted model has lower

values in the low frequencies but has higher values in the high frequencies than those of the unweighted case. Particularly, both models display nominal stability since they satisfy  $\rho(E_m \tilde{H}) < 1, \forall \omega$  condition. A comparison of open-loop step responses displays the difference in gain values between the unweighted and weighted MFD models and the plant (Figure 6). In closed-loop MPC setpoint tracking test, the weighted model has no offset in  $y_1$  and  $y_2$  while the unweighted suffers offset in  $y_1$  and more oscillation in  $y_2$  (Figure 7).

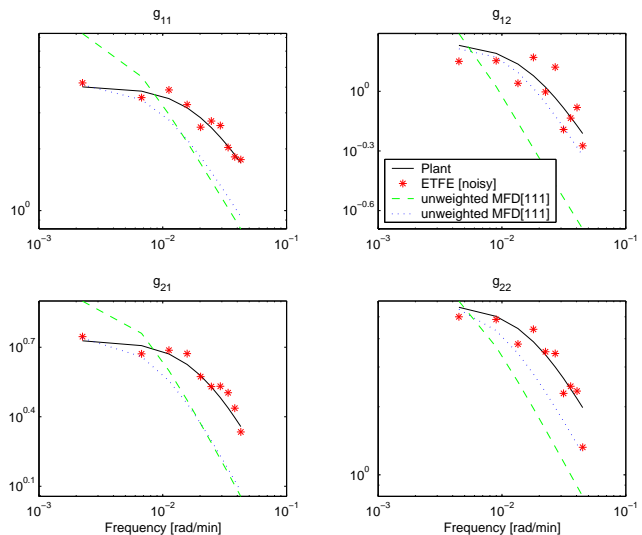


Fig. 4. Frequency response curvefitting with MFD[ $n_a = 1, n_b = 1, n_d = 1$ ] models per (32) under noisy conditions,  $\sigma_d^2 = 2.0$  (solid: true plant, \*: ETFE values, dashed: unweighted MFD, dotted: weighted MFD)

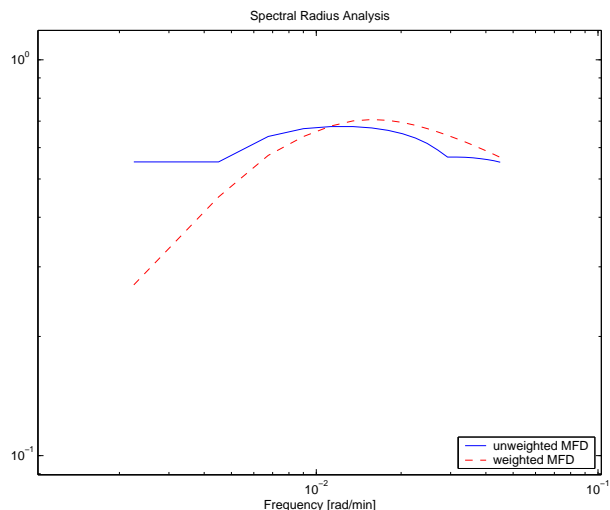


Fig. 5.  $\rho(E_m \tilde{H})$  comparison between the unweighted and weighted MFD[ $n_a = 1, n_b = 1, n_d = 1$ ] models per (32)

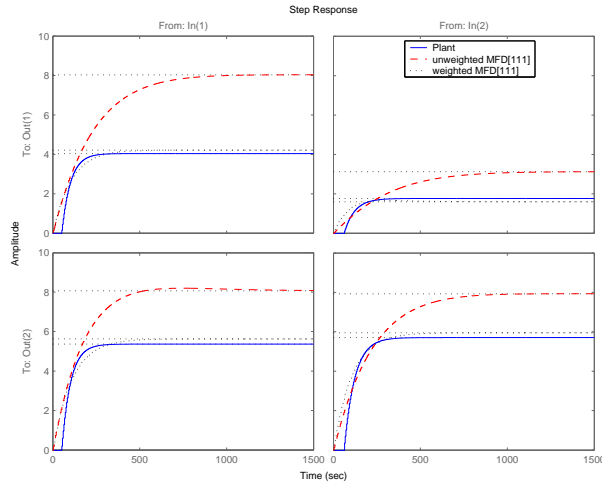


Fig. 6. Open-loop step responses of MFD[ $n_a = 1, n_b = 1, n_d = 1$ ] models per (32)(solid: true plant, dashed: unweighted MFD, dotted: weighted MFD)

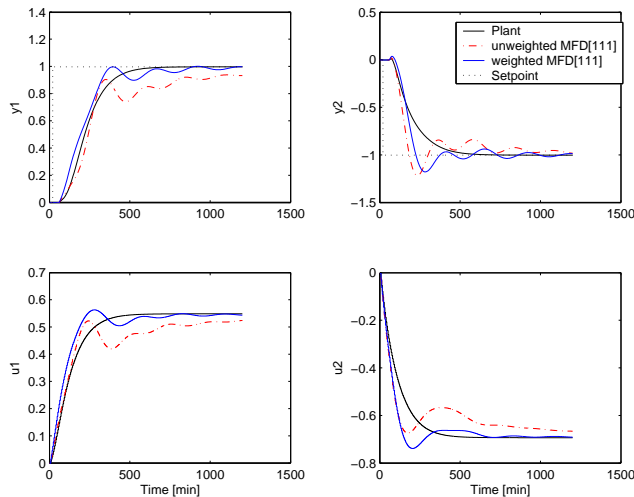


Fig. 7. Closed-loop MPC setpoint tracking test using MFD[ $n_a = 1, n_b = 1, n_d = 1$ ] models per 32: PH=50, MH=10, Ywt=[1 1], & Uwt=[32 40] (solid: true plant, dashed: unweighted MFD, dash-dotted: weighted MFD, dotted: setpoint)

## VII. CONCLUSIONS

In this paper, a method for parametric model estimation from frequency-weighted curvefitting is achieved by the use of the multisine input signals and the full-polynomial MFD approach. This MFD is implemented to support the plant-friendly multisine inputs based on zippered frequency grids with possible harmonic suppression. The objective function of the CRPEP is numerically solved by iterative minimization procedures using S-K iteration and GN-optimization.

In the example case study, the weighted curvefitting shows the frequency-dependent error minimization utilizing the control-relevant pre/post-weights as functions of

the closed-loop dynamics. Since the weighting functions emphasize the low-frequency dynamics in the example, the parameter estimation error is shifted into the high frequency area that is less relevant to the closed-loop control performance. Therefore, the control-relevant weights properly reflect the closed-loop dynamics in the curvefitting of frequency responses into linear MFD models. Future research considers integrating this work into a comprehensive identification test monitoring procedure [10].

## VIII. ACKNOWLEDGEMENT

This research has been supported by the American Chemical Society - Petroleum Research Fund, Grant No. ACS PRF# 37610-AC9.

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