

CONTROL-RELEVANT DEMAND MODELING FOR SUPPLY CHAIN MANAGEMENT

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Abstract: The development of control-oriented decision policies for inventory management in supply chains has received considerable interest in recent years, and demand modeling to supply forecasts for these policies is an important component of an effective solution to this problem. Drawing from the problem of control-relevant identification, we present an approach for demand modeling based on data that relies on a control-relevant prefilter to tailor the emphasis of the fit to the intended purpose of the model, which is to provide forecast signals to a tactical inventory management policy based on Model Predictive Control. Integrating the demand modeling and inventory control problems offers the opportunity to obtain reduced-order models that exhibit superior performance, with potentially lower user effort relative to traditional “open-loop” methods. A systematic approach to generating these prefilters is presented and the benefits resulting from their use are demonstrated on a representative production/inventory system case study.

Keywords: Control-relevant identification, supply chain management, demand modeling, model reduction

1. INTRODUCTION

Efficient supply chain management has become a significant imperative for many modern-day enterprises (Simchi-Levi *et al.*, 2004). Properly characterizing and predicting demand plays a significant role in achieving high performance from supply chain management systems. The presence of error in a demand forecast will adversely affect decision-making in a supply chain. Inaccurate market research, customer order changes, out-of-date information, and misreading product/business cycles may have a negative effect on the profitability of a supply chain dependent corporation. While eliminating all sources of error from a demand forecast

is impossible, it may be possible to mitigate its detrimental effects. Therefore, it is important to understand the effects of forecast error on a supply chain decision policy.

Control-oriented approaches have been recently proposed to deal with the inventory management problems inherent in supply chains (Tzafestas *et al.*, 1997; Dejonckheere *et al.*, 2002; Perea-López *et al.*, 2003; Braun *et al.*, 2003; Seferlis and Giannelos, 2004; Wang *et al.*, 2004). In these approaches, demand is treated as an exogenous “disturbance” signal that must be properly “rejected” by a sensibly-designed control system. However, an understanding of how a demand forecast should be properly developed for the sake of this class of supply chain management policies has not been examined. This paper attempts to gain a broader understanding of disturbance/demand

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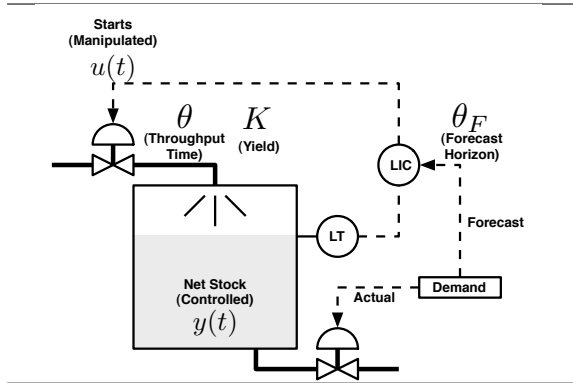


Fig. 1. Fluid analogy: a single manufacturing node represented as a system of pipes and a tank.

modeling and the effects of forecast error on a Model Predictive Control (MPC)-based tactical decision policy. The relationship between demand forecast error and changes in inventory and starts are examined for a control-oriented tactical decision policy in a single node of the manufacturing process. Understanding this relationship represents one step towards a fundamental understanding that will allow planning personnel to deal with inherently erroneous forecasts in an educated manner.

To accomplish this goal, we will draw from ideas in control-relevant identification (Rivera *et al.*, 1992). The result is a systematic framework for conducting control-relevant demand modeling in the single variable case. Results from a case study will show that the use of the framework not only improves the performance of the supply chain, but also enables planners to reduce the complexity of customer demand models.

Section 2 begins with a discussion of the modeling of a production/inventory system using a fluid analogy and the development of a model-based inventory controller relying on Model Predictive Control. In Section 3, the closed-loop transfer functions describing forecast error are developed and the effect of erroneous forecasts is studied in both the time and frequency domains. A procedure for performing control-relevant demand modeling is presented. Section 4 is a case study involving the use of an MPC scheme to manage a production/inventory system. Section 5 highlights the important conclusions that can be drawn from the analysis in this paper.

2. SYSTEM AND CONTROLLER

2.1 Inventory Control Fluid Analogy

A single node of a manufacturing supply chain can be modeled using a fluid analogy. The factory is represented as a pipe with a particular throughput time (θ) and yield (K). The inventory

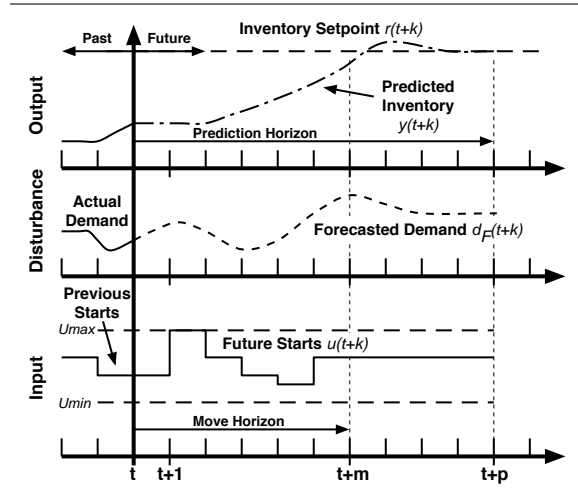


Fig. 2. Receding horizon representation of Model Predictive Control.

is represented as a tank containing fluid. The dynamics relating fluid level (net stock, $y(t)$) to inlet pipe flux (fab starts, $u(t)$) and outlet pipe flux ($d(t)$), composed of the forecasted customer demand, $d_F(t - \theta_F)$, plus unforecasted customer demand, $d_U(t)$ is represented in (1). Note that θ_F is the forecast horizon. The underlying dynamical system has delayed, integrating dynamics according to

$$y(t) = \frac{Kz^{-\theta} u(t)}{1 - z^{-1}} - \frac{z^{-\theta_F} d_F(t)}{1 - z^{-1}} - \frac{d_U(t)}{1 - z^{-1}} \quad (1)$$

The operational goal of the system is to meet customer demand while maintaining the inventory level at a specified target. This can be accomplished by adjusting the factory starts. An anticipated (forecasted) demand signal can be used for feedforward compensation in this regard.

2.2 Model Predictive Control

Model Predictive Control (MPC) (García *et al.*, 1989; Camacho and Bordons, 1999) stands for a family of methods that select control actions based on on-line optimization of an objective function. In MPC, a system model and current and historical measurements of the process are used to predict the system behavior at future time instants. A control-relevant objective function is then optimized to calculate a sequence of future control moves that satisfy system constraints. The first predicted control move is implemented and at the next sampling time the calculations are repeated using updated system states; this is referred to as a Moving or Receding Horizon strategy. Fig. 2 is a useful visualization of the MPC approach. The demand signal (which dictates the shipment of product to the customer) consists of two components: 1) actual demand (which is only

fully known as it occurs) and 2) forecasted demand, which is provided to the planning function by a separate organization. As shown in Fig. 2, a demand forecast signal is used in the moving horizon calculation to anticipate future system behavior, which plays a significant role in the use of MPC for supply chain applications.

As shown in Eqn. 2, the Model Predictive Control strategy relies on a state-space form of Eqn. 1 to make predictions of the future output (inventory level) and adjust the input (factory starts) according to the current state of the system and a forecast of future disturbances (customer demand).

$$\begin{aligned} x(t+1) &= A x(t) + B_u u(t) + B_d d(t) \\ y(t) &= C x(t) + D_u u(t) + D_d d(t) \end{aligned} \quad (2)$$

y , u , and d are as defined previously, $x(t)$ is the state vector and A , B_u , B_d , D_u , and D_d represent constant-valued matrices.

There is significant flexibility in the form of the objective function that can be used in MPC. The formulation considered in this paper is to minimize the following:

$$\begin{aligned} J &= \min_{\Delta u(k|k) \dots \Delta u(k+m-1|k)} J \\ &= \sum_{\ell=1}^p Q_e (\hat{y}(k+\ell|k) - r(k+\ell))^2 \\ &\quad + \sum_{\ell=1}^m Q_{\Delta u} (\Delta u(k+\ell-1|k))^2 \end{aligned} \quad (3)$$

subject to constraints on inventory capacity ($0 \leq y(t) \leq y_{max}$), factory inflow capacity ($0 \leq u(t) \leq u_{max}$), and changes in the quantity of factory starts ($\Delta u_{min} \leq \Delta u \leq \Delta u_{max}$). The objective function is a multi-objective expression that addresses the main operational objectives in the supply chain. For an MPC problem with an objective function per (3), relying on linear discrete-time state-space models to describe the dynamics, and subject to linear inequality constraints, a numerical solution is achieved via a quadratic program.

3. CONTROL-RELEVANT DEMAND MODELING

3.1 Achieving a Frequency-Domain Understanding of the Effects of Forecast Error

Typically, system identification models are generated via the minimization of the one-step ahead prediction error (Ljung, 1999). This classical approach does not take into consideration the end use of the model, such as when the goal of the model is to support a control-oriented decision

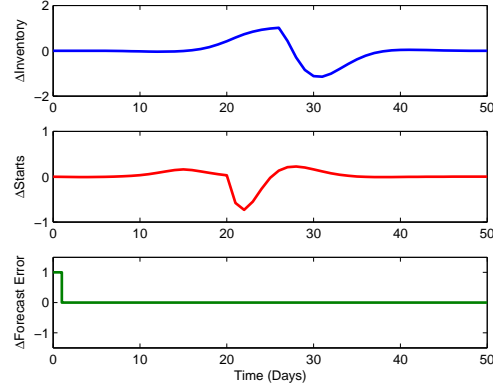


Fig. 3. Supply chain control system response to a unit forecast error pulse. Throughput time: $\theta = 5$ days, prediction horizon: p and $\theta_F = 20$ days, move horizon: $m = 10$ days, move suppression: $Q_{\Delta u} = 5$, control error weight: $Q_e = 1$.

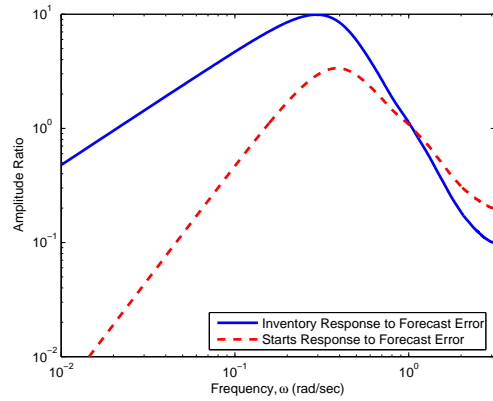


Fig. 4. Amplitude ratio of the inventory and starts response to forecast error ($\frac{y(e^{j\omega})}{e_F(e^{j\omega})}$ and $\frac{u(e^{j\omega})}{e_F(e^{j\omega})}$, respectively) for the model and controller parameters per Fig. 3.

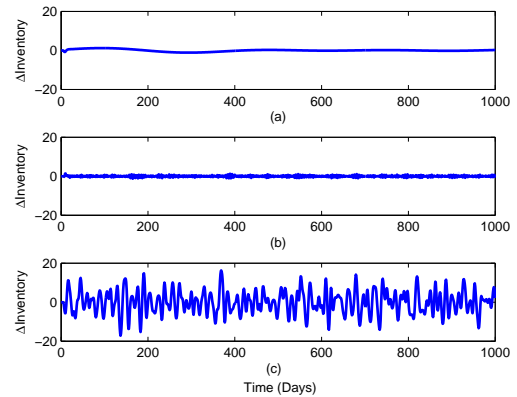


Fig. 5. Inventory response to low frequency (a), high frequency (b), and intermediate frequency forecast error (c).

policy. In particular, we are interested in understanding how forecast error affects the performance of the closed-loop system. It is our contention in this paper that such systems are most responsive to forecast error within a certain frequency bandwidth. Subsequently, the performance of the control system can be improved by utilizing demand models that are most accurate within the frequency band of interest, resulting in the potential for arriving at better demand models with less effort, which has important practical implications.

Given that unconstrained MPC is a linear control system and that linearity is assumed for the production/inventory system, the frequency response of the closed-loop system can be completely characterized via nonparametric methods. Fig. 3 shows some representative results of the time-domain inventory and starts responses to a forecast error impulse. The controller anticipates the increased future demand and increases starts accordingly. When no demand change is realized, starts are decreased to return the inventory level to the setpoint. These responses can be captured as Finite Impulse Response models, from which frequency responses are generated. Fig. 4 shows the corresponding amplitude ratios of the MPC closed-loop system.

Fig. 4 shows that the response of the MPC decision policy to forecast error is characterized by a notch filter, where high and low frequencies are attenuated, and only forecast error in an intermediate bandwidth is amplified (the size of which is determined by controller tuning). This is illustrated in the time-domain in Fig. 5 where bandlimited forecast error signals with unity variance are introduced. Forecast error in the intermediate bandwidth causes the most change in the inventory, and it is in this bandwidth that a high degree of goodness-of-fit in the demand model is desired.

3.2 Control-Relevant Modeling

Since the decision policy under study is known to amplify forecast error present in a limited frequency bandwidth, it is desirable to systematically take advantage of the reduced bandwidth over which demand modeling accuracy is necessary. Therefore, the goal is to emphasize the frequencies of interest when generating a model. Prefiltering represents an important design variable for emphasizing the goodness-of-fit in system identification (Ljung, 1999; Rivera *et al.*, 1992). Assume that true demand is described by a stationary process $p_d(z)$ driven by the input signal $u_d(t)$ plus some unforecasted component $H(z)a(t)$. For the purposes of this analysis, the input signal $u_d(t)$ is known; in a univariate case

this signal can be reconstructed using a two-stage approach (Stoica and Moses, 1997). In practical applications, the input signal could represent variables that influence demand such as interest rates, seasonal changes, and per capita income.

$$d(t) = p_d(z)u_d(t) + H(z)a(t) \quad (4)$$

Our goal is to find a model $\tilde{p}_d(z)$ for the true process $p_d(z)$. For the purposes of this analysis the demand is defined as the sum of the contributions from the transfer function $p_d(z)$ and a noise model $\tilde{p}_e(z)$.

$$d(t) = \tilde{p}_d(z)u_d(t) + \tilde{p}_e(z)e(t) \quad (5)$$

$e(t)$ is defined as the one-step ahead prediction error. Typical system identification problems would then involve the minimization of the filtered one-step ahead prediction error, where $L(z)$ is the prefilter.

$$\min_{\tilde{p}_d} V = \min_{\tilde{p}_d} \sum_{t=1}^N [L(z)e(t)]^2 = \min_{\tilde{p}_d} \sum_{t=1}^N e_L^2(t) \quad (6)$$

The filtered prediction error is comprised of:

$$e_L(t) = \frac{L(z)}{\tilde{p}_e(z)} [(p_d(z) - \tilde{p}_d(z))u_d(t) + H(z)a(t)] \quad (7)$$

The application of Parseval's theorem allows an analysis of the problem in the frequency domain.

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^N e_L^2(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{L(e^{j\omega})}{\tilde{p}_e(e^{j\omega})} \right|^2 \left(|p_d(e^{j\omega}) - \tilde{p}_d(e^{j\omega})|^2 \Phi_{u_d}(\omega) + |H(e^{j\omega})|^2 \Phi_a(\omega) \right) d\omega \quad (8)$$

From Eqn. 8 we know that we can use $L(z)$ to provide user-defined emphasis, although it is important to note that the spectrum of the input signal Φ_{u_d} and the noise model $\tilde{p}_e(z)$ also act to emphasize certain frequency regimes. The user choice of the Output Error structure results in $\tilde{p}_e(z) = 1$, eliminating the bias introduced by the noise model. Our goal is to obtain estimates $\tilde{p}_d(z)$ of the true demand process $p_d(z)$ with emphasis in the frequencies of interest defined by the control-relevant prefilter $L(z)$, where $\frac{|L(e^{j\omega})|}{|\tilde{p}_e(e^{j\omega})|} \Phi_a^{\frac{1}{2}}(\omega)$ matches the weighting implied by the control-relevant analysis of Section 3.1 and illustrated in Fig. 4. The resulting increase in goodness-of-fit in the intermediate frequency bandwidth will improve control system performance and allow the complexity of demand models to be reduced.

4. CASE STUDY

A representative production/inventory system with an MPC-based tactical decision policy will be used to quantify the benefits achieved through the use of control-relevant demand modeling. The case study involves the single node shown in Fig. 1 where the throughput time of the factory (θ) is 5 days, the forecast horizon (θ_F , which is also the MPC prediction horizon p) is 20 days, the MPC move optimization horizon (m) is 10 days, and MPC weights for penalizing starts changes and inventory deviation ($Q_{\Delta u}$ and Q_e) are 5 and 1, respectively. A data set was generated from the true demand process $p_d(z)$ subject to a white noise input. This model (a 10th order Auto Regressive Moving Average model) is characterized by unity gain and significant power at high frequencies, as shown in Fig. 6. The amplitude ratio of the control-relevant prefilter $L(e^{j\omega})$ is also shown. The demand model conforms to an Output Error (OE-[n_b n_f 1]) structure as shown in Eqn. 9.

$$\hat{d}(t) = \frac{b_1 + \dots + b_{n_b} z^{-n_b+1}}{1 + \dots + f_{n_f} z^{-n_f}} u(t-1) + e(t) \quad (9)$$

Fig. 7 shows the inventory (a) and starts variance (b) that occur when estimating demand models of varying complexity ($1 \leq n_b \leq 10$ and $0 \leq n_f \leq 10$). The number of OE parameters is defined as the sum of n_b and n_f . Low order models (up to five parameters) obtained from prefiltered data provide superior performance relative to the classical unfiltered approach. Note that while inventory variation is decreased through the use of the control-relevant demand forecast, starts variance may increase. If a supply chain operator's objective is to minimize factory thrash (starts variance) a different filter (such as the starts frequency response shown in Fig. 4) could be used.

Fig. 6 shows the amplitude ratios of the OE-[1 2 1] models developed from raw $\tilde{p}_d^{(raw)}(e^{j\omega})$ and control-relevant filtered $\tilde{p}_d^{(c-r)}(e^{j\omega})$ data. The lack of emphasis in the unfiltered data causes the OE fit to focus on the high frequency spike at approximately 3 radians per second. The control-relevant filter emphasizes the resonant peak while de-emphasizing the high frequency components of the demand. Consequently, the OE fit to the control-relevant data captures the dynamics of the true demand spectrum where the closed-loop system is most sensitive to forecast error. Fig. 8 shows time-series results corresponding to the two examples. For this particular low-order model structure the use of the control-relevant prefilter leads to significantly lower inventory and starts variance.

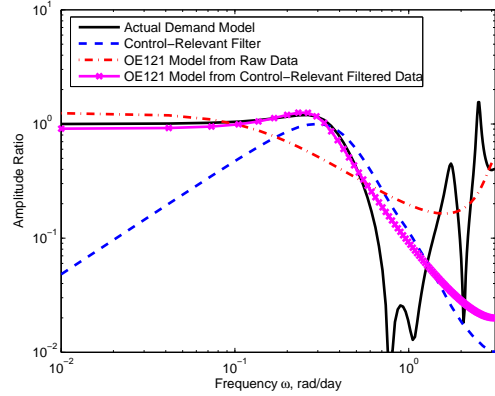


Fig. 6. Amplitude ratios of the actual demand model ($p_d(e^{j\omega})$, solid line), control-relevant prefilter ($L(e^{j\omega})$, dashed line), OE-[1 2 1] fit to raw data ($\tilde{p}_d^{(raw)}(e^{j\omega})$, dashed-dot line), and OE-[1 2 1] fit to control-relevant data ($\tilde{p}_d^{(c-r)}(e^{j\omega})$, crossed line).

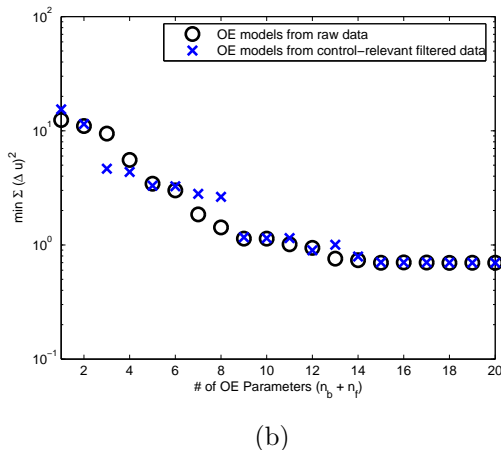
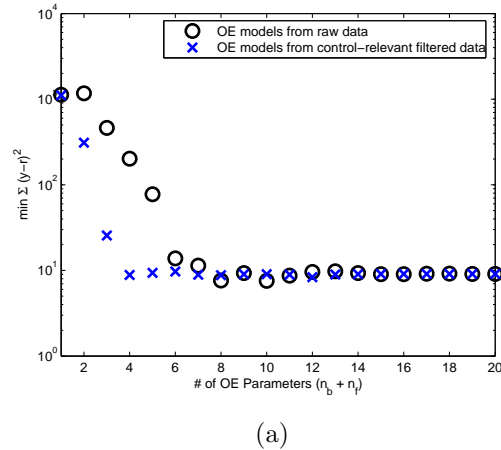
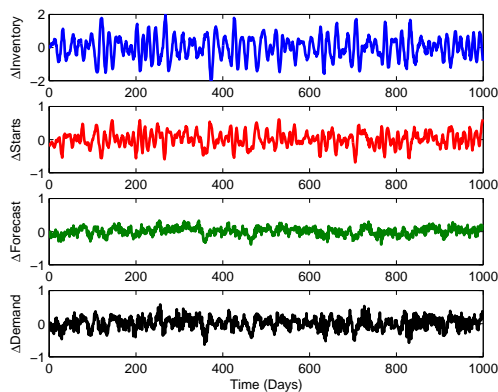
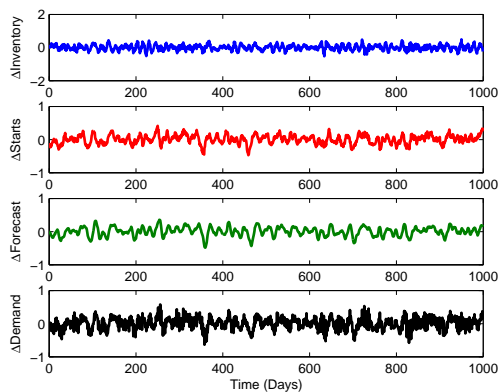


Fig. 7. Lowest inventory variance (a) and starts variance (b) for a variety of estimated OE demand models. The plot shows the best performance for the group of models defined as having $n_b + n_f$ parameters.



(a)



(b)

Fig. 8. Time series where the demand forecast is developed from (a) an OE-[1 2 1] fit to raw data ($\Sigma(y - r)^2 = 475$, $\Sigma(\Delta u)^2 = 9$) and (b) an OE-[1 2 1] fit to control-relevant filtered data ($\Sigma(y - r)^2 = 26$, $\Sigma(\Delta u)^2 = 5$).

5. CONCLUSIONS

Demand modeling is a critical problem in supply chain management. An analysis of an MPC-based decision policy associated with inventory control shows that these systems are most responsive to forecast error in an intermediate frequency bandwidth. With this knowledge, historical demand data can be prefiltered to emphasize the frequency regimes where the most accuracy is desired, resulting in improved control system performance. The work also shows that the use of control-relevant prefiltering allows simpler, low-order models to be used. This results in more efficient computation and greater insight into the most relevant characteristics associated with customer demand.

Future work will involve extending the analysis to forecasting problems involving Box-Jenkins-style approaches (Box *et al.*, 1994), and multivariable demand modeling problems.

6. ACKNOWLEDGEMENTS

The authors would like to acknowledge support from the Intel Research Council and the National Science Foundation (DMI-0432439).

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